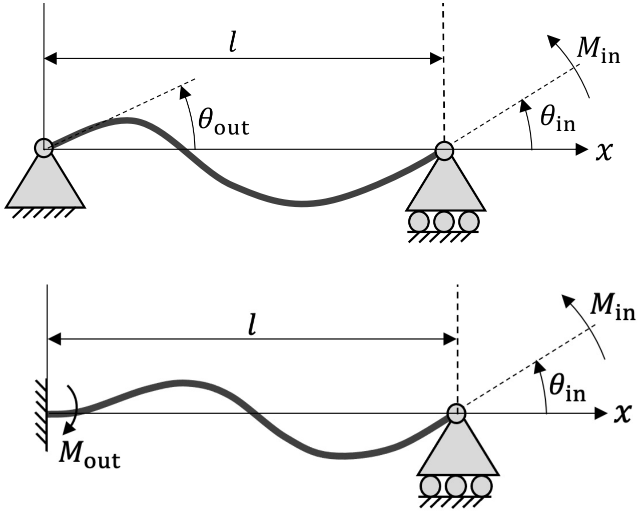
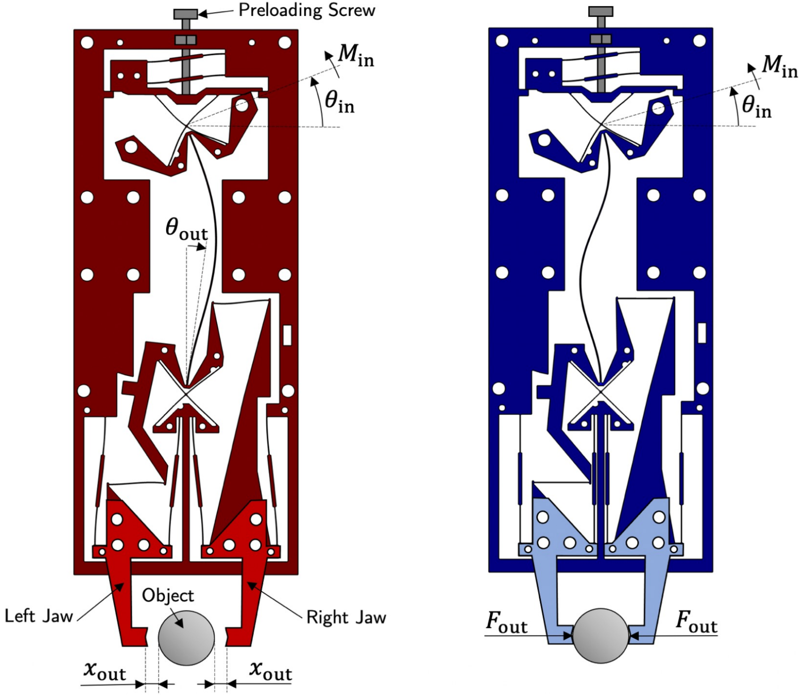
Flexure mechanisms with post-buckled beams

Loïc Tissot-Daguette, Simon Henein
Instant-Lab





EPFL

May 2025

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Introduction

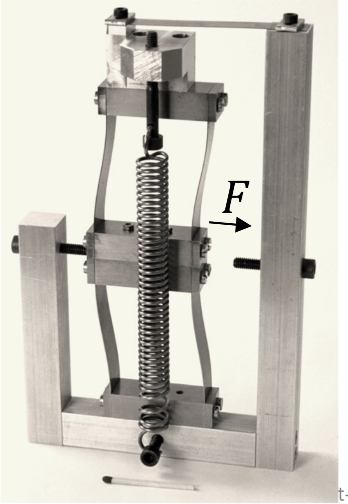
Buckled Beams

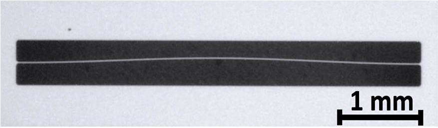
Civil Engineering:
"Failure by Buckling" ✗

External Load

Internal Load

Mechanical Engineering:
"Stiffness Modification" ✓





(Silicon beam buckled by thermal oxidation)

Buckled beams as mechanism components

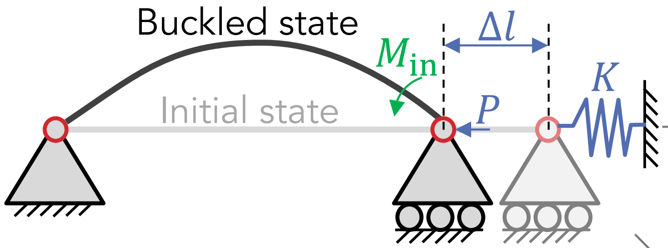
Mechanical functions:

Buckled Beams

- ✓ **Bistability**
- ✓ Energy Storage/Release
- ✓ Snap-Through
- ✓ **Stiffness Reduction**
- ✓ Static Balancing
- ✓ Negative Stiffness
- ✓ **Constant-Force**
- ✓ Force Limitation

Buckled beams as mechanism components

<u>Load cases:</u>



Boundary Conditions
- Fixed-fixed
- Fixed-guided
- Pinned-pinned
- Fixed-pinned
- Fixed-lever
- …

Preloading
- Type I: Geometric ($\Delta l = cst$)
- Type II: Elastic ($\Delta l = \frac{P_0 - P}{K}$)
- Type III: Multistable ($\Delta l_1, \Delta l_2, \ldots$)
- Type IV: Self-preloading
- Type V: Residual stress

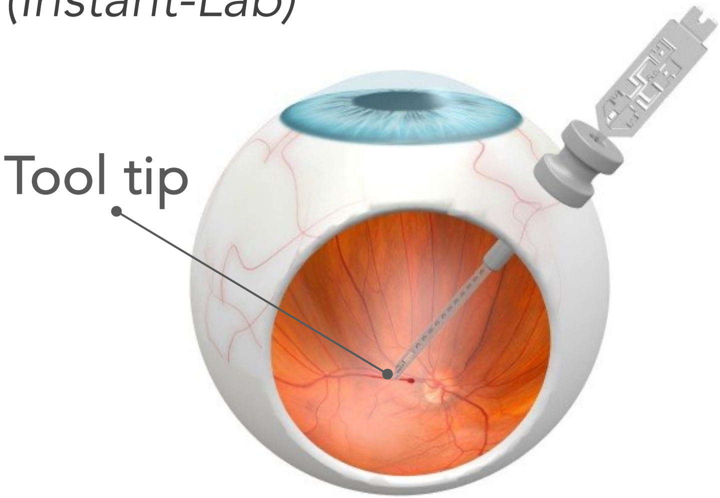
Actuation
- Force/Translation
- Moment/Rotation

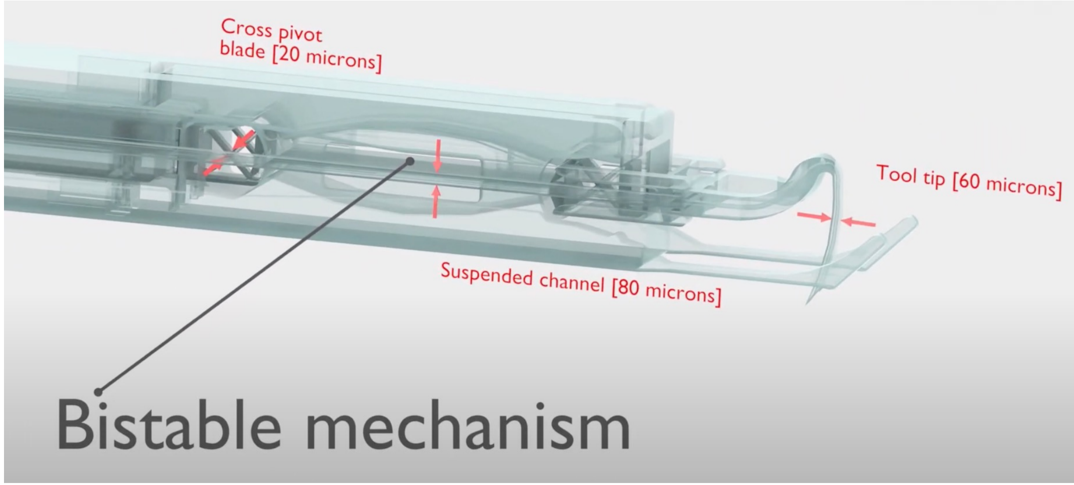
State of the art

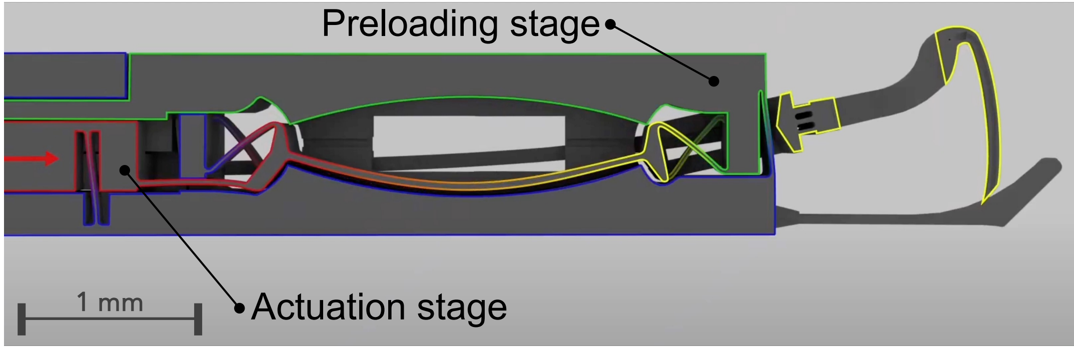
Preloading Type I: *Geometric*

Safe Puncture Optimized Tool for Retinal Vein Cannulation

(Instant-Lab)







M. Zanaty et al., **Programmable Multistable Mechanisms for Safe Surgical Puncturing,** ASME. J. Med. Devices, Volume 13 (2), 2019

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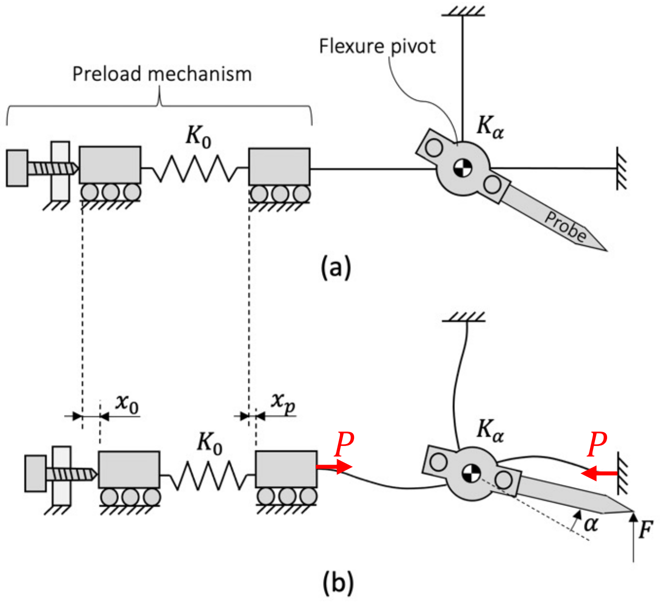
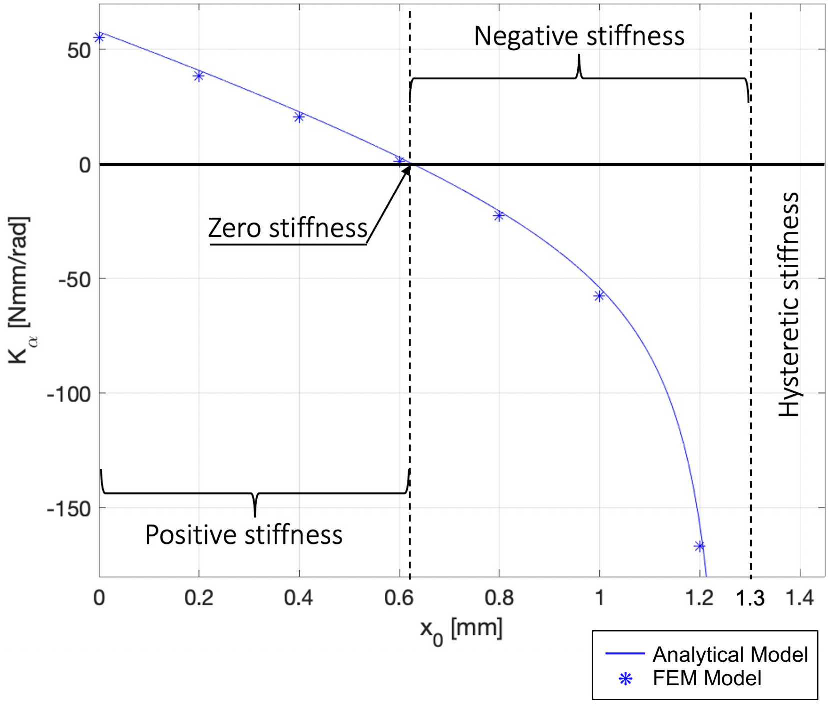
Preloading Type II: *Elastic*

Load Cell with Stiffness Tuning *(Instant-Lab)*





L. Tissot-Daguette et al., **Load cell with adjustable stiffness based on a preloaded T-shaped flexure pivot**, in: Euspen's 21st International Conference & Exhibition, 2021

State of the art

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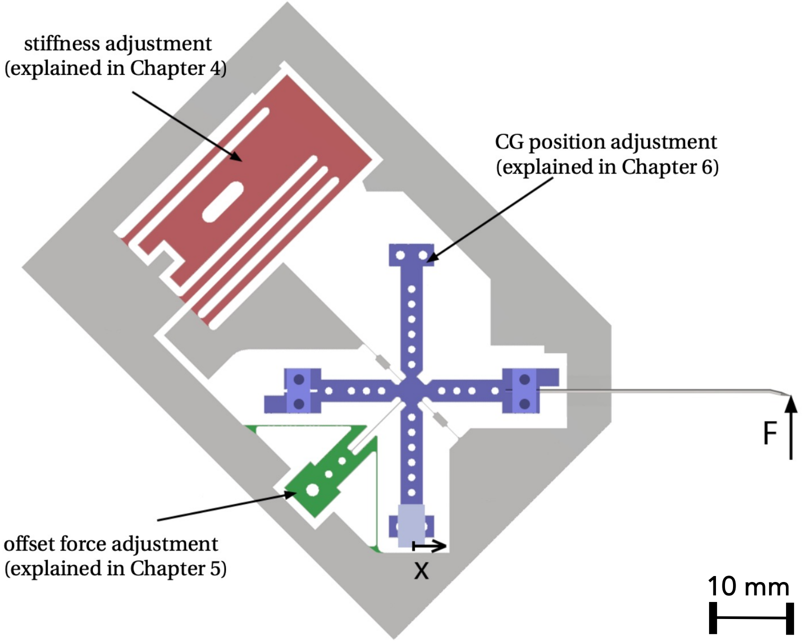
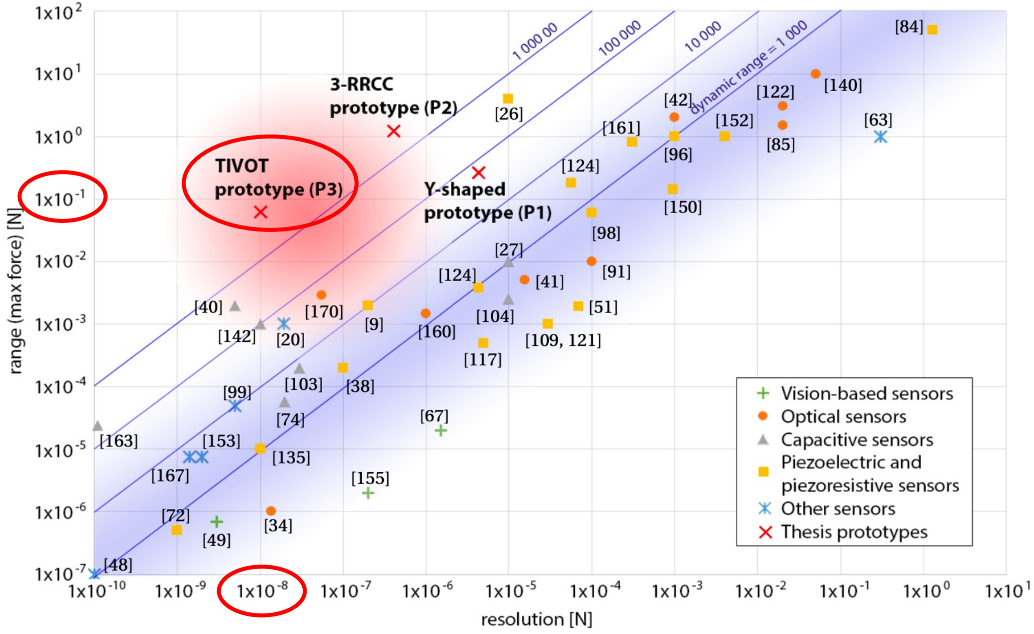
Load Cell with Stiffness Tuning *(Instant-Lab)*





M. Smreczak et al., **A load cell with adjustable stiffness and zero offset tuning dedicated to electrical micro- and nanoprobing**, Precision Engineering, Volume 76, 208-225, 2022

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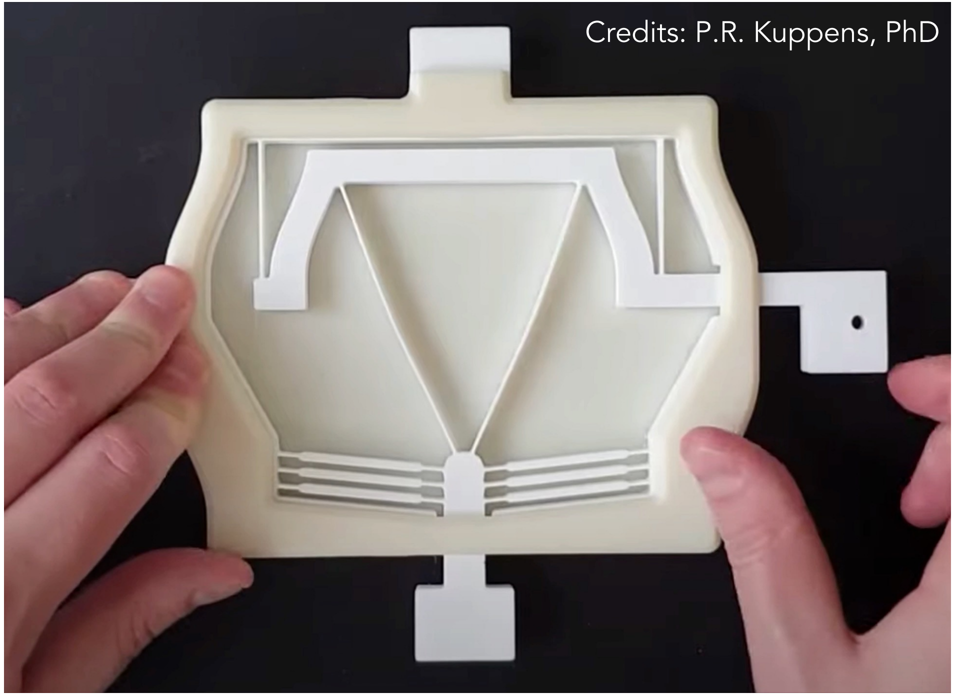
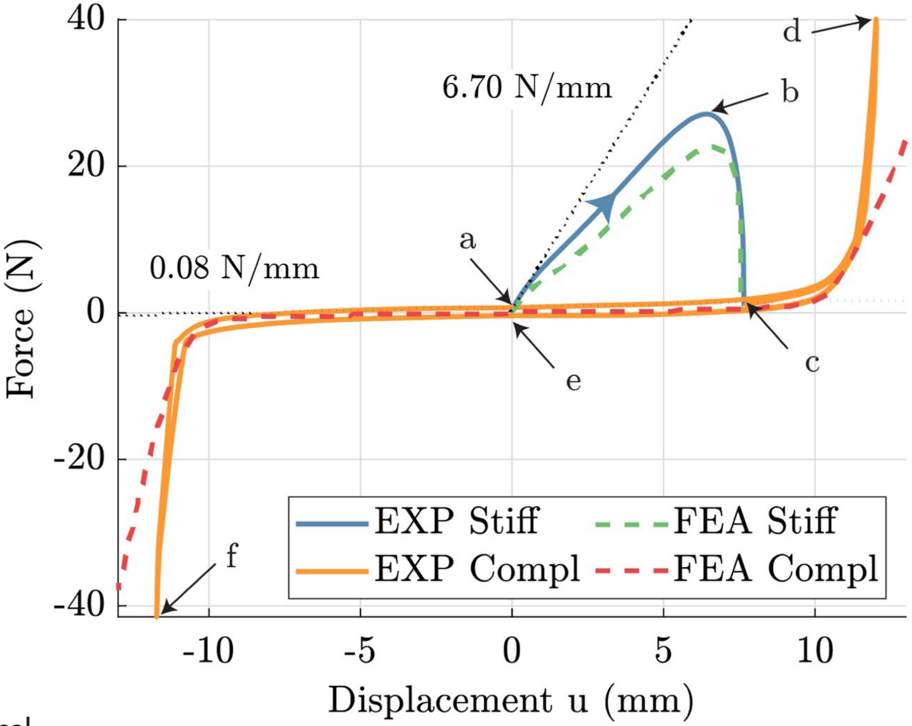
Preloading Type III:
Multi-stable

Binary Stiffness Compliant Mechanism





P.R. Kuppens et al., **Monolithic binary stiffness building blocks for mechanical digital machines,** Extreme Mechanics Letters, Volume 42, 101120, 2021

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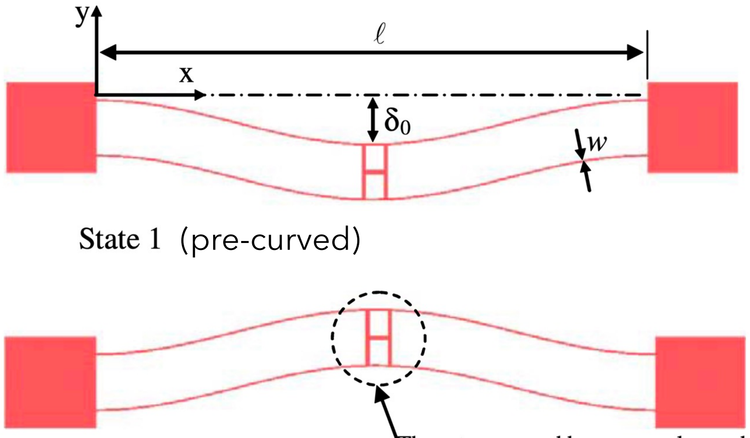
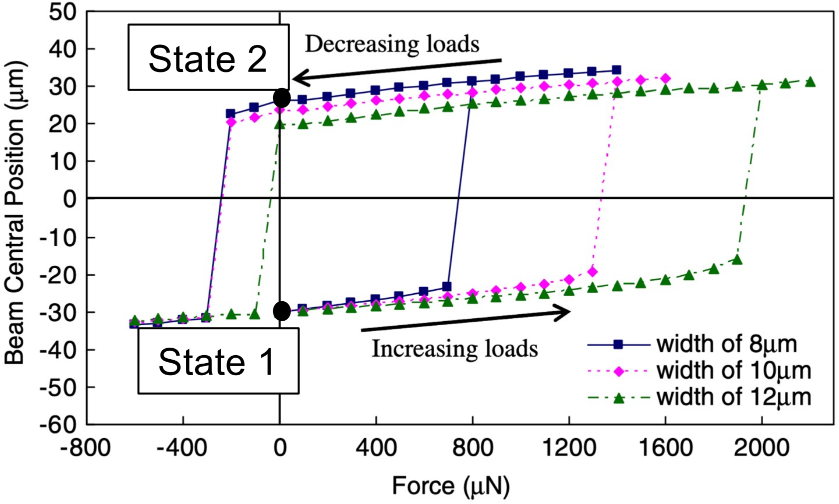
Preloading Type IV:
Self-preloading

Bistable MEMS Optical Switch



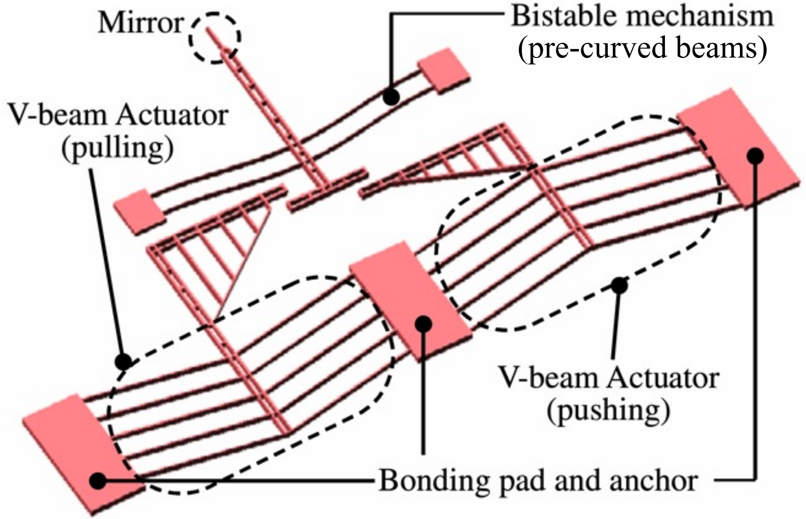


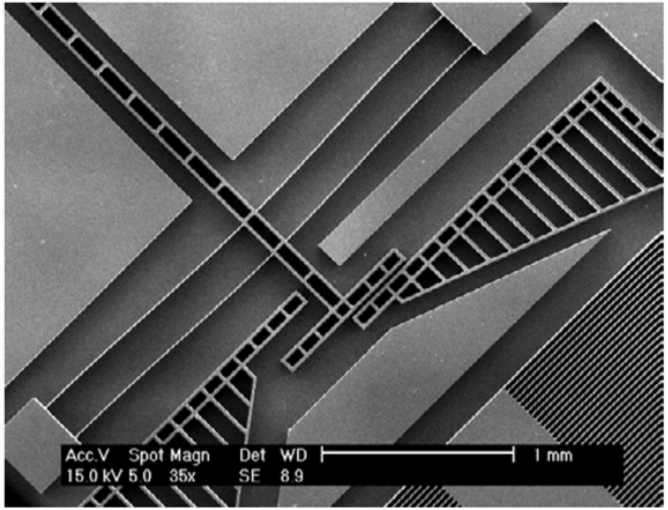
Y.-J. Yang et al., **A novel 2 × 2 MEMS optical switch using the split cross-bar design**, J. Micromech. and Microeng., Volume 17, 875-882, 2007

State of the art

Preloading Type IV:
Self-preloading

Bistable MEMS Optical Switch



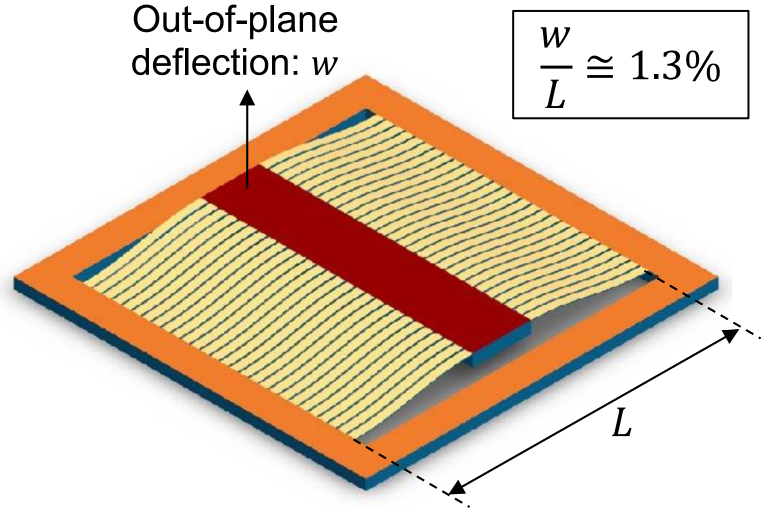


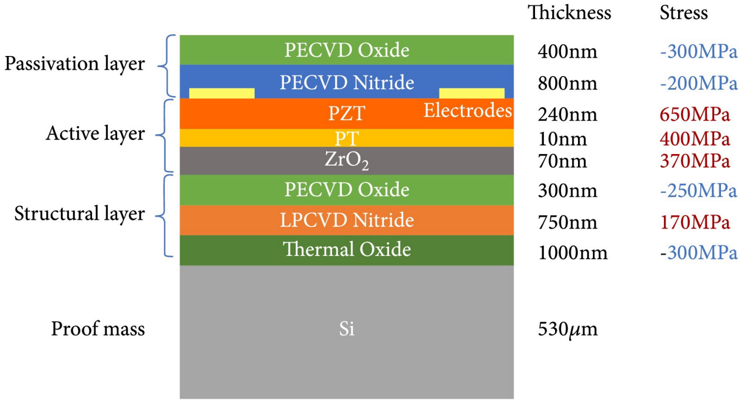
Y.-J. Yang et al., **A novel 2 × 2 MEMS optical switch using the split cross-bar design**, J. Micromech. and Microeng. 17 (2007) 875–882

State of the art

Preloading Type V: *Residual stress*

Bistable MEMS Energy Harvester





X. Ruize et al., **Buckled MEMS Beams for Energy Harvesting from Low Frequency Vibrations**, Research. 2019 (2019)

Generic analytical model

- Euler-Bernoulli beam theory with the linearization $x \cong \frac{l}{L} s$:

$$M(s) = -Py(s) + V\frac{l}{L}s + M_0 \cong EI\frac{d^2y(s)}{ds^2}$$

- General solution of the beam deflection:

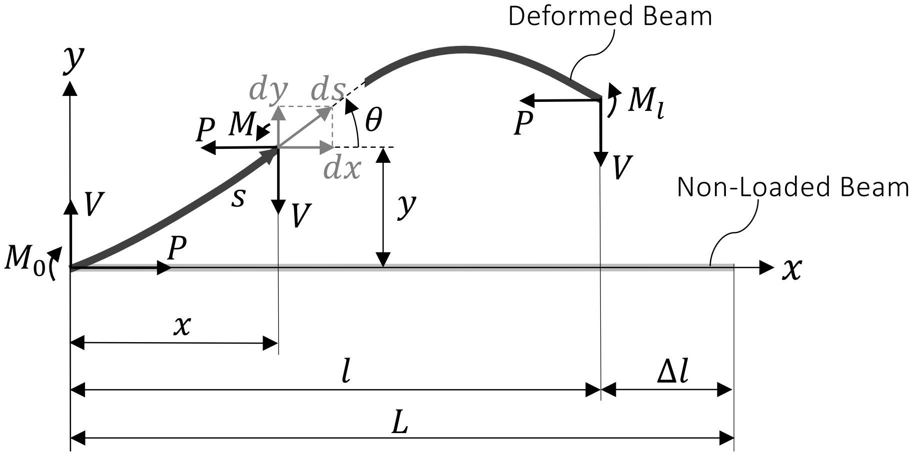
$$y(x) = A\sin\left(kL\frac{x}{l}\right) + B\left(\cos\left(kL\frac{x}{l}\right) - 1\right) + CL\frac{x}{l}$$

where: $k = \sqrt{\frac{P}{EI}}$

- Beam shortening constraint:

$$\Delta l = L - l \cong \frac{1}{2}\int_0^L \left(\frac{dy}{ds}\right)^2 ds$$

$$\Delta l = \frac{(A^2+B^2)(kL)^2}{4L} + \frac{(A^2-B^2)kL\sin(2kL)}{8L} + \frac{ABkL(\cos(2kL)-1)}{4L} + \frac{C^2L}{2} + AC\sin(kL) + BC(\cos(kL)-1)$$



Deflection of a generic compressed beam

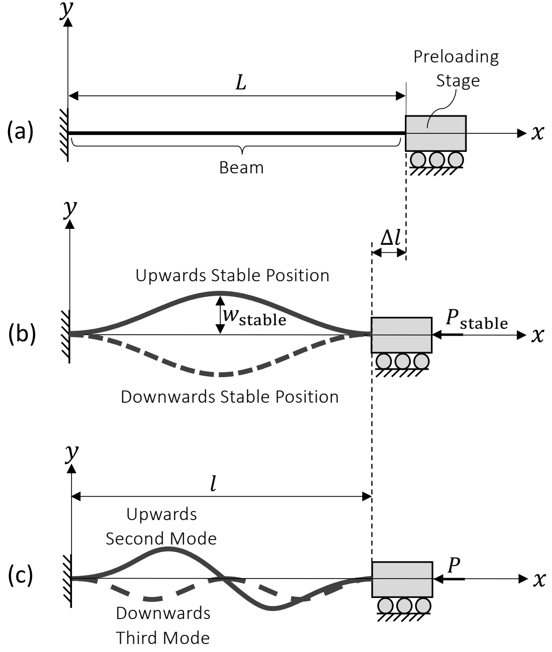
- L. Tissot-Daguette et al., **Analytical modeling and experimental validation of rotationally actuated pinned–pinned and fixed–pinned buckled beam bistable mechanisms**, Mech. Mach. Theory, 174, 104874 (2022)

Behavior of some load cases

Fixed-fixed buckled beam:

Preloading Type IV: *Self-Preloading*

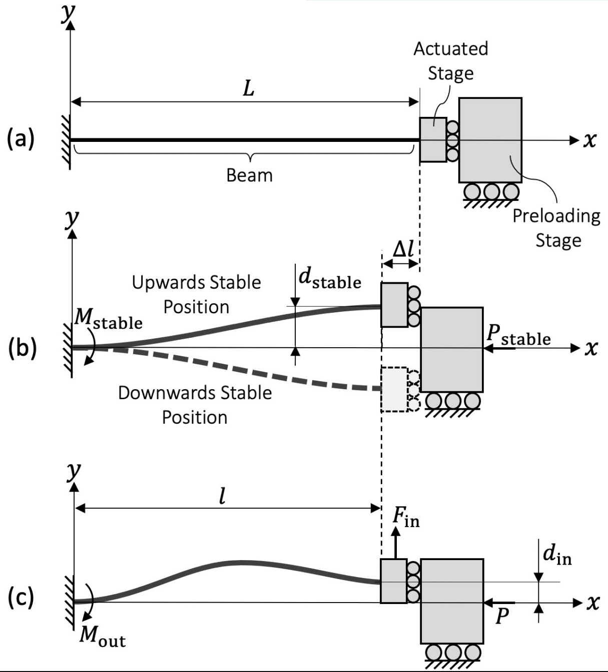
Constant-force behavior!



Fixed-guided buckled beam:

Preloading Type I: *Geometric*

Negative stiffness behavior!



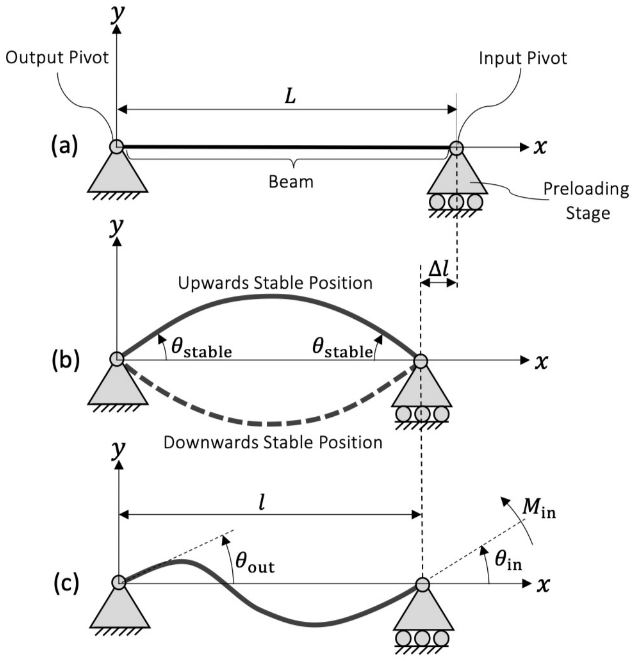
Behavior of some load cases

Pinned-pinned buckled beam:	Fixed-pinned buckled beam:

Pinned-pinned buckled beam:

Preloading Type I: *Geometric*

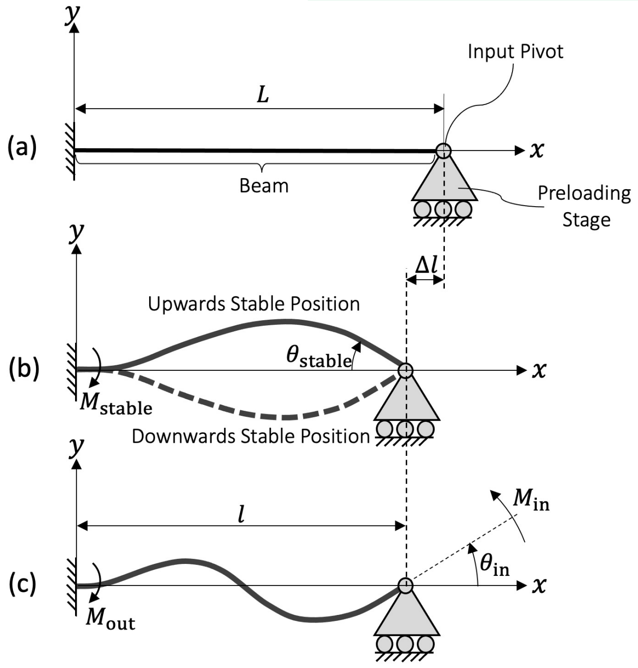
Bistable and snap-through behaviors!



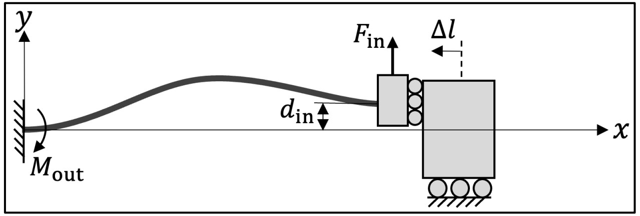
Fixed-pinned buckled beam:

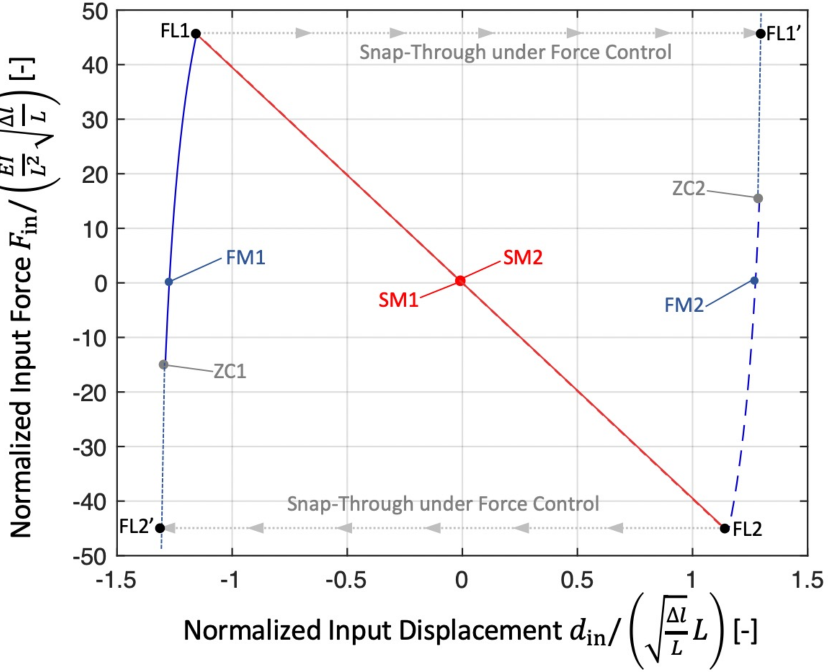
Preloading Type I: *Geometric*

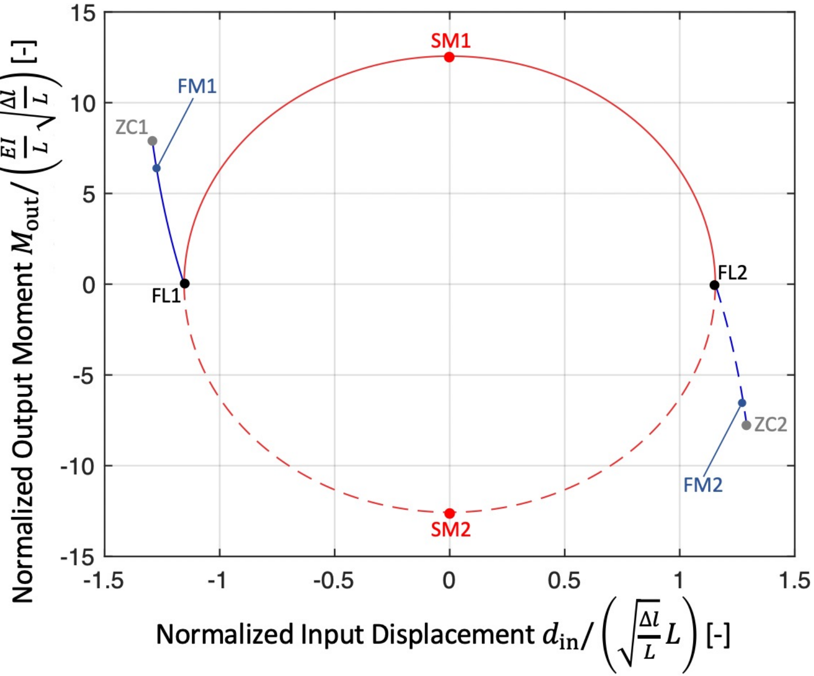
Bistable and snap-through behaviors!



Modeling: Basic load cases







Branches:

- First Mode Branch 1
- Second Mode Branch 1
- First Mode Branch 2
- Second Mode Branch 2

Points:

FM = First Mode

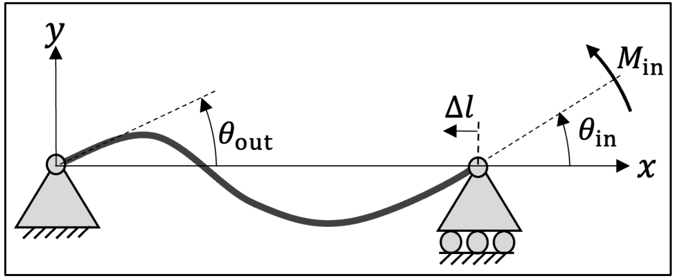
SM = Second Mode

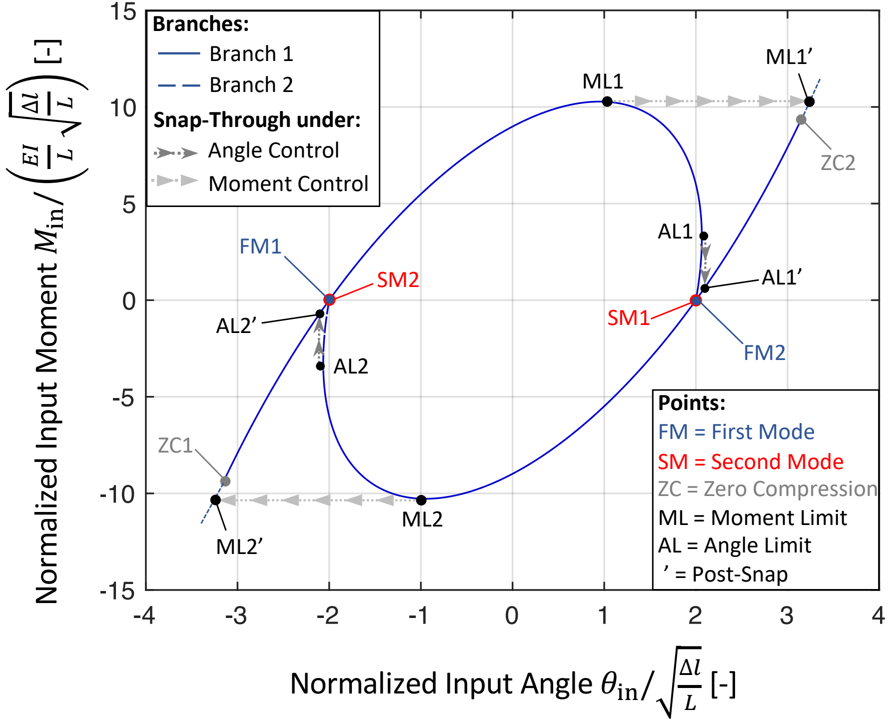
ZC = Zero Compression

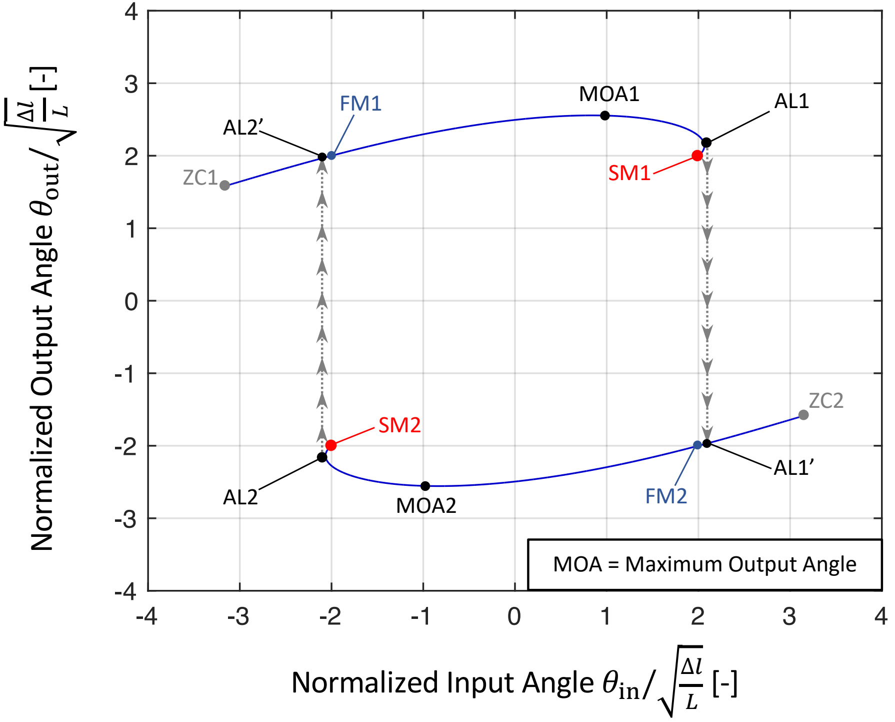
FL = Force Limit

FL' = Post-Snap Position

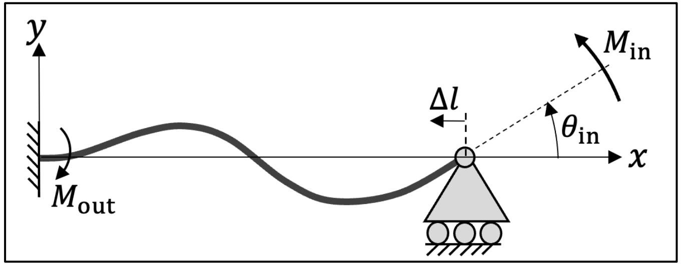
Modeling: Basic load cases

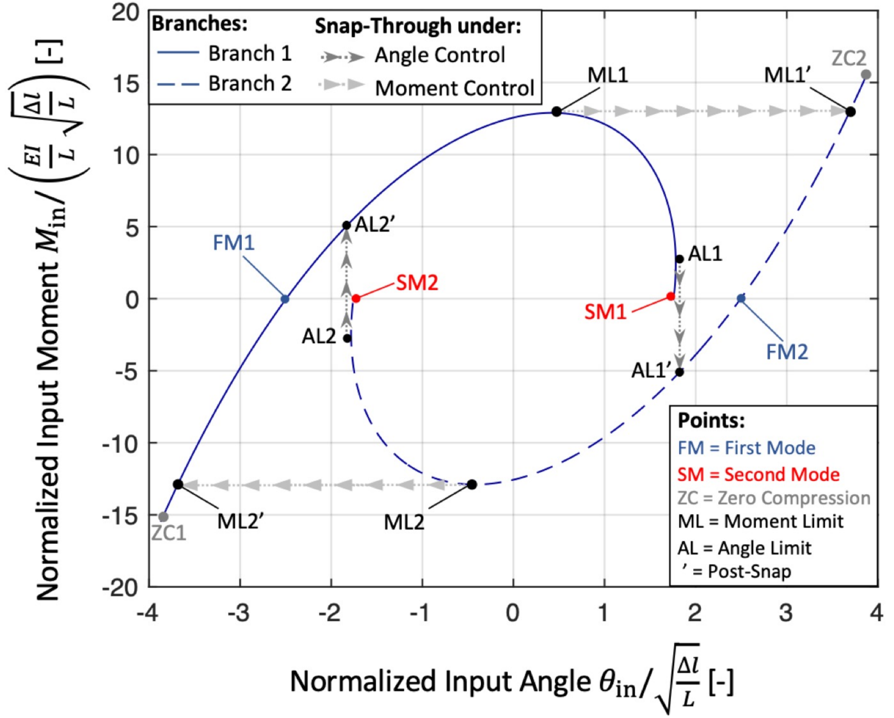


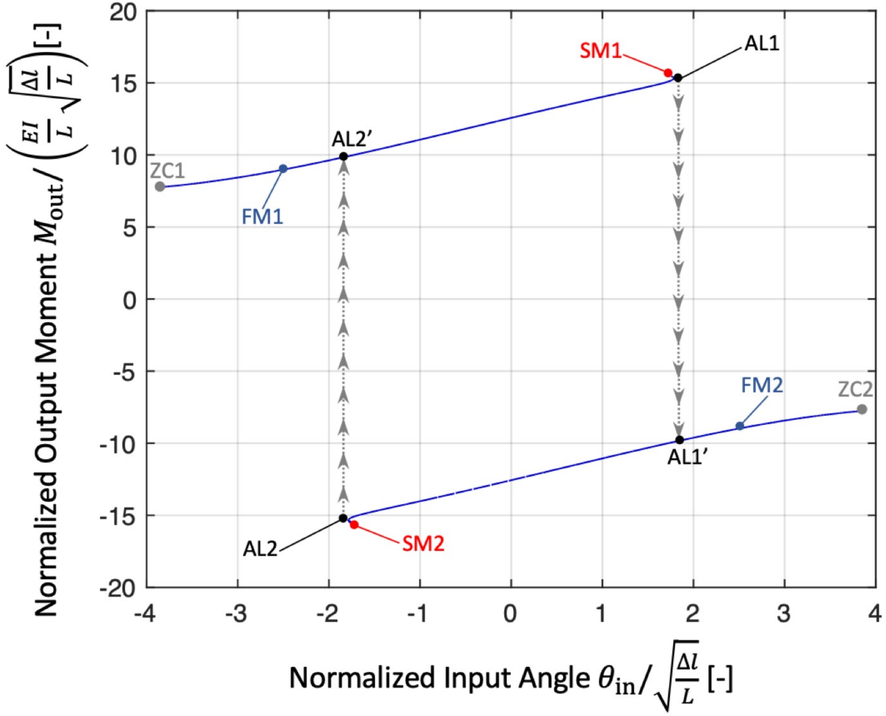




Modeling: Basic load cases

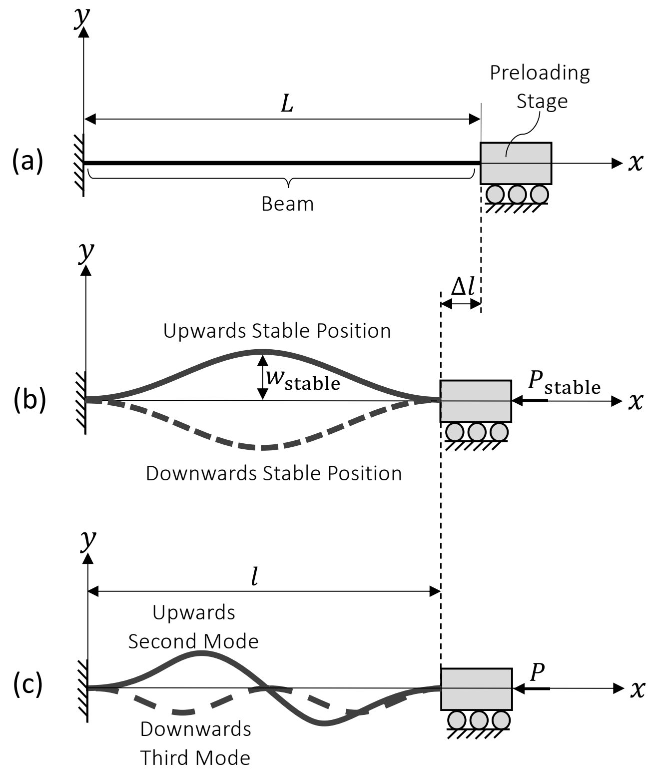






Modeling: Formulas

Fixed-fixed buckled beam:



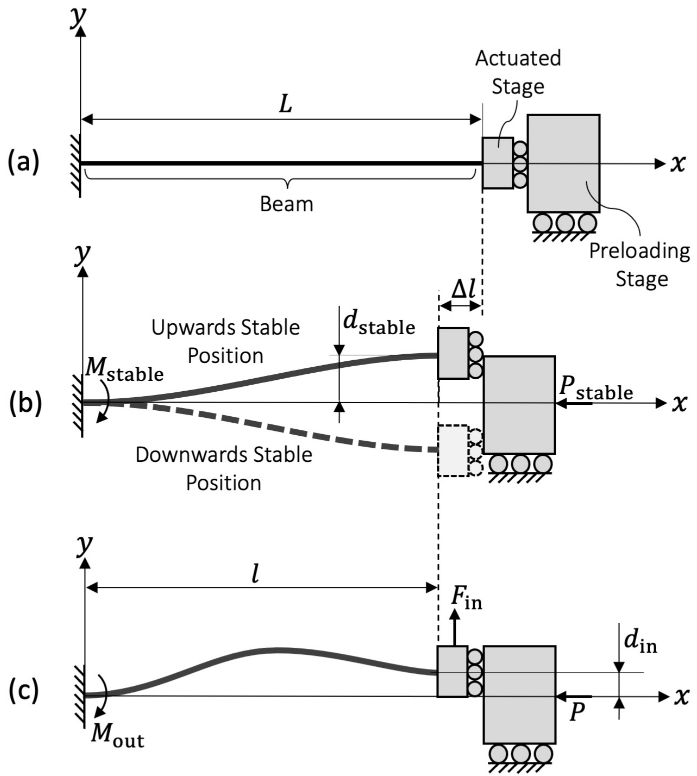
- First mode positions (stable states):

$$w_{\text{stable}} = \frac{2}{\pi}\sqrt{\frac{\Delta l}{L}}\,L, \qquad P_{\text{stable}} = \frac{4\pi^2 EI}{L^2},$$

$$M_{\text{stable}} = \pm\frac{4\pi EI}{L}\sqrt{\frac{\Delta l}{L}}$$

Modeling: Formulas

Fixed-guided buckled beam:



- First mode positions (stable states):

$$d_{\text{stable}} = \frac{4}{\pi}\sqrt{\frac{\Delta l}{L}}L, \qquad P_{\text{stable}} = \frac{\pi^2 EI}{L^2},$$

$$M_{\text{stable}} = \frac{2\pi EI}{L}\sqrt{\frac{\Delta l}{L}}$$

- Second mode branch (constant negative stiffness region):

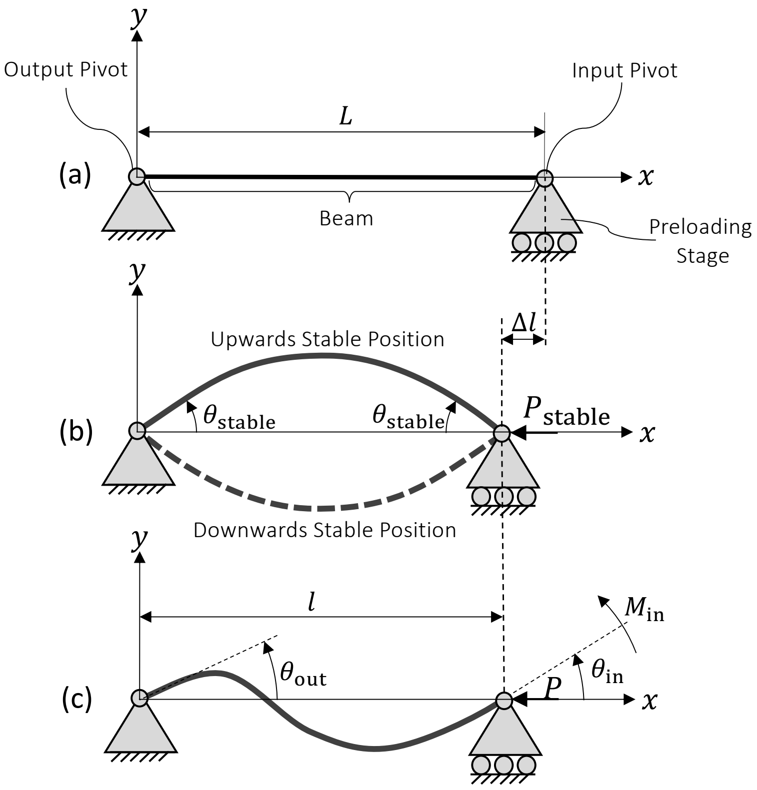
$$K_{\text{in}} = \frac{F_{\text{in}}}{d_{\text{in}}} = -\frac{4\pi^2 EI}{L^3}, \qquad P = \frac{4\pi^2 EI}{L^2},$$

$$M_{\text{out}} = \pm\frac{2\pi EI}{L}\sqrt{4\frac{\Delta l}{L} - 3\left(\frac{d_{\text{in}}}{L}\right)^2},$$

where: $-d_{\text{in,max}} < d_{\text{in}} < d_{\text{in,max}}, \quad d_{\text{in,max}} = \frac{2}{\sqrt{3}}\sqrt{\frac{\Delta l}{L}}L$

Modeling: Formulas

Pinned-pinned buckled beam:



- First mode positions (stable states):

$$\theta_{\text{stable}} = 2\sqrt{\frac{\Delta l}{L}}, \qquad P_{\text{stable}} = \frac{\pi^2 EI}{L^2}$$

- Limit point magnitudes:

$$M_{\text{in,lim}} = 10.28\frac{EI}{L}\sqrt{\frac{\Delta l}{L}}$$

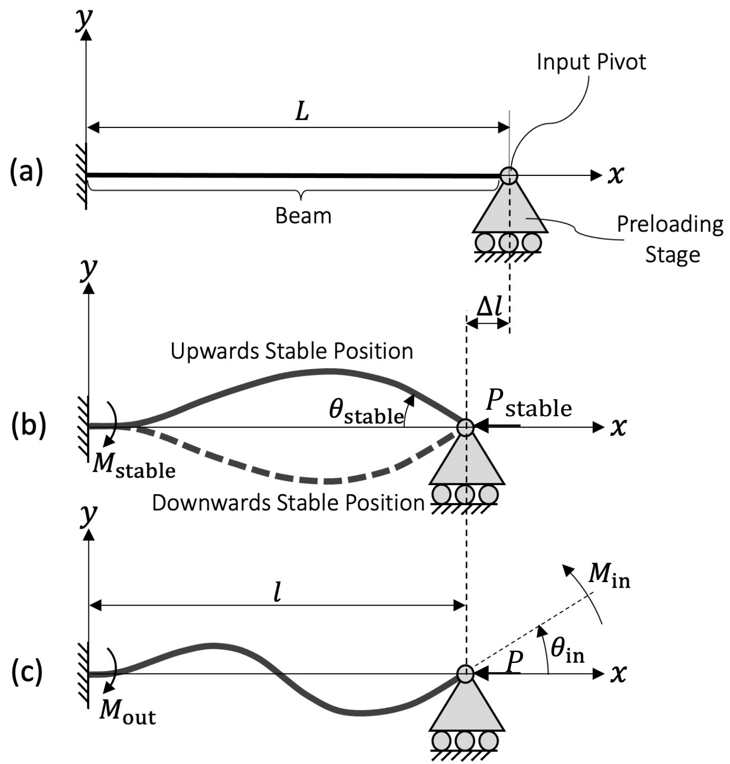
$$\theta_{\text{in,lim}} = 2.07\sqrt{\frac{\Delta l}{L}}$$

- Output maximum magnitude:

$$\theta_{\text{out,max}} = 2.56\sqrt{\frac{\Delta l}{L}}$$

Modeling: Formulas

Fixed-pinned buckled beam:



- First mode positions (stable states):

$$\theta_{\text{stable}} = 2.49\sqrt{\frac{\Delta l}{L}}, \qquad P_{\text{stable}} = \frac{(4.49)^2 EI}{L^2},$$

$$M_{\text{stable}} = 8.99\,\frac{EI}{L}\sqrt{\frac{\Delta l}{L}}$$

- Limit point magnitudes:

$$M_{\text{in,lim}} = 12.90\,\frac{EI}{L}\sqrt{\frac{\Delta l}{L}}$$

$$\theta_{\text{in,lim}} = 1.78\sqrt{\frac{\Delta l}{L}}$$

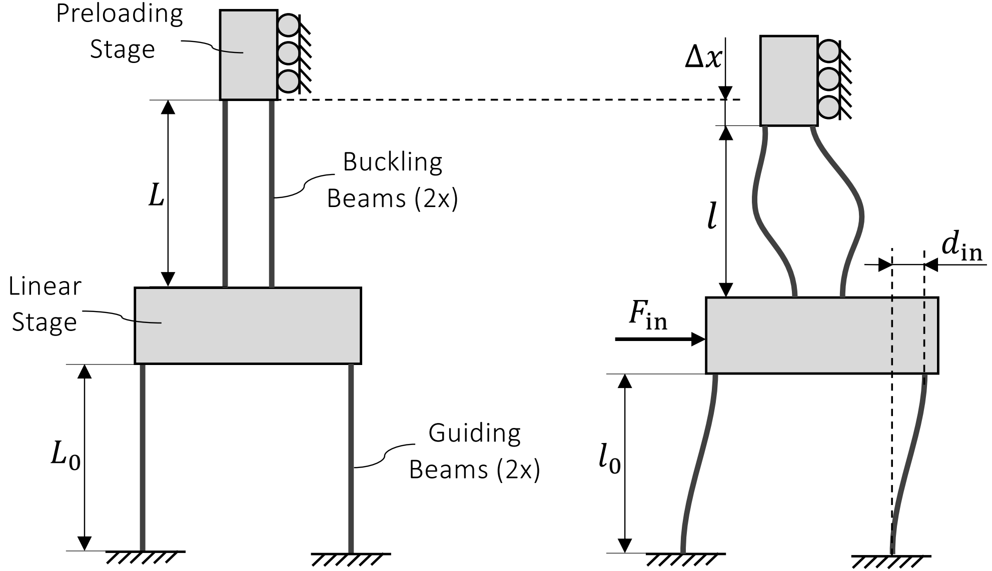
- Output maximum magnitude:

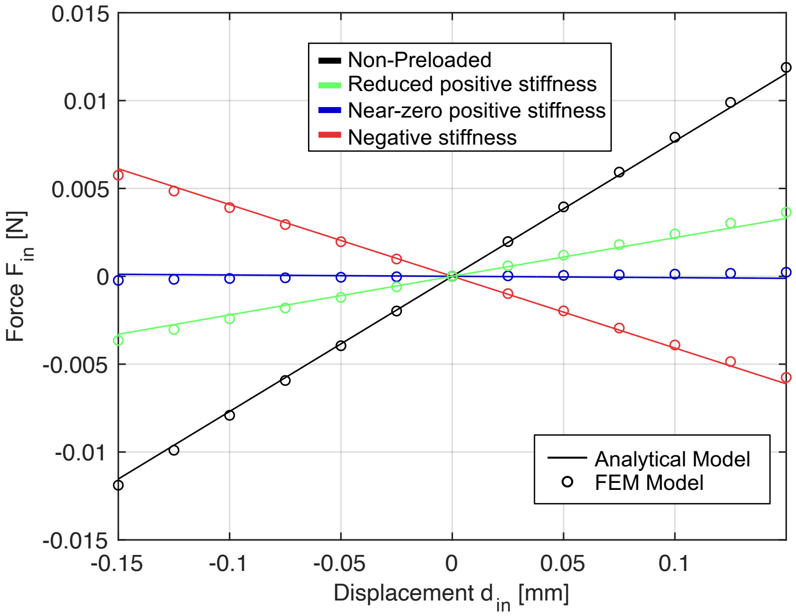
$$M_{\text{out,max}} = 15.31\,\frac{EI}{L}\sqrt{\frac{\Delta l}{L}}$$

Elementary mechanisms

Stiffness-reduced linear stage:

Preloading Type I:
Geometric ($\Delta x = cst$)



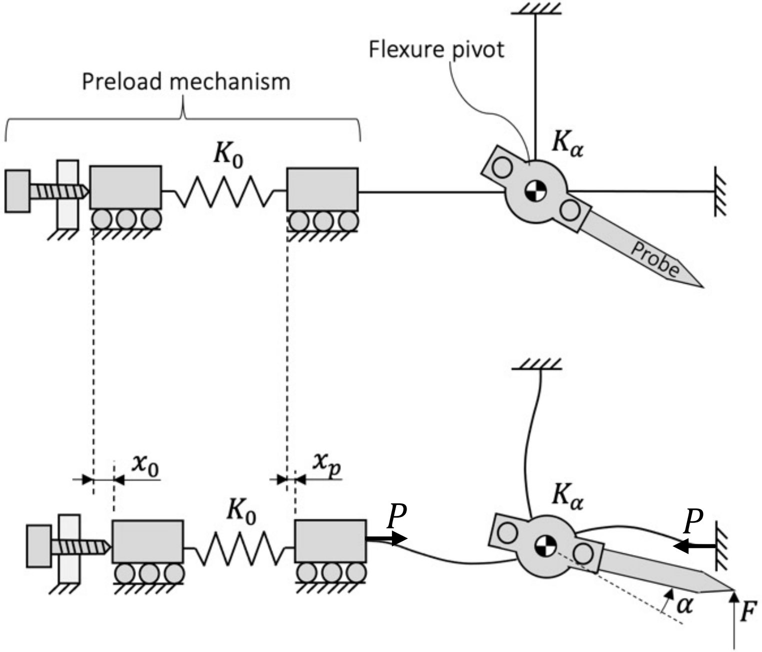


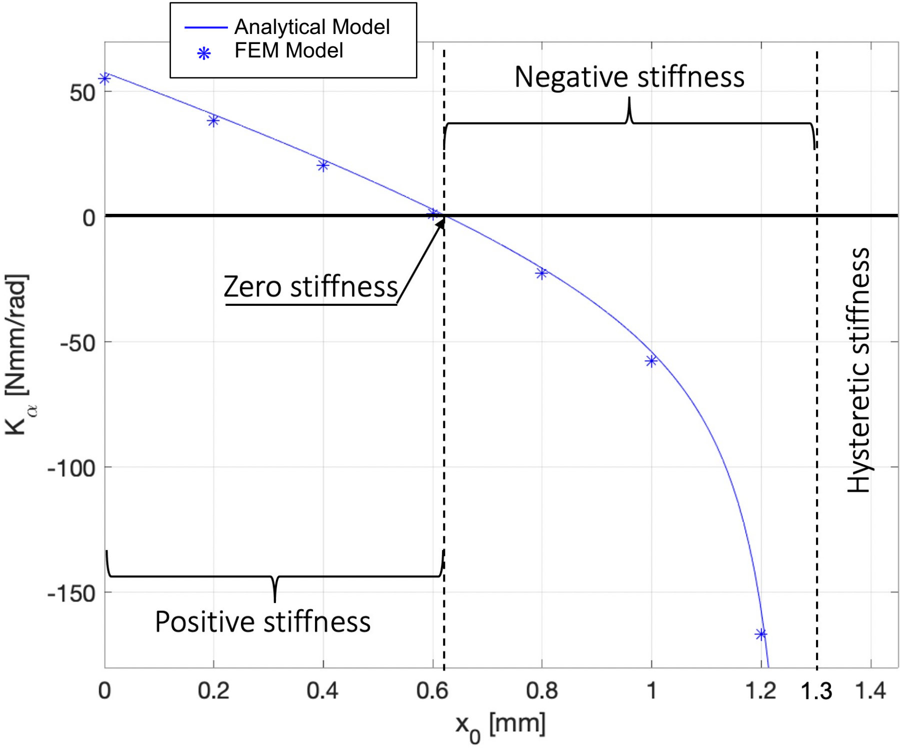
- L. Tissot-Daguette et al., **Residual Stress Chevron Preloading Amplifier for Large-Stroke Stiffness Reduction of Silicon Flexure Mechanisms**, J. Micromech. Microeng. 35, 025003 (2025)

Pivot with adjustable stiffness:

Preloading Type II: *Elastic*

Patented & Published



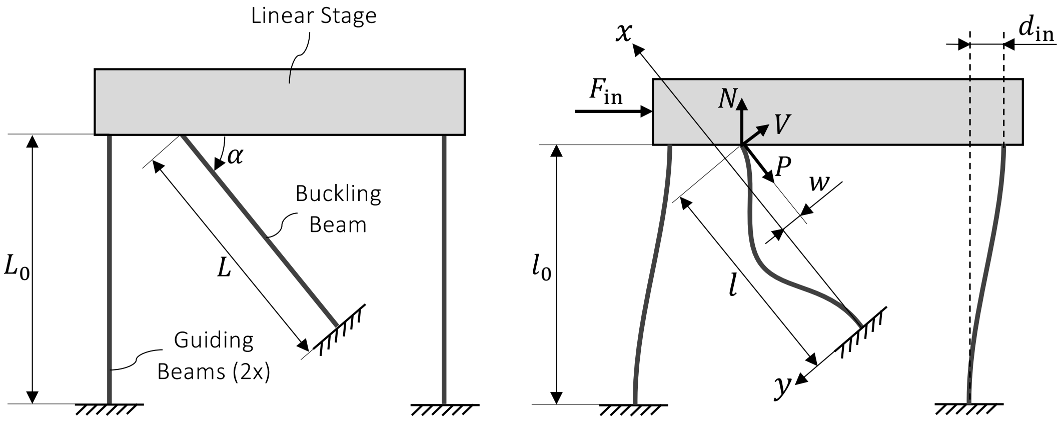


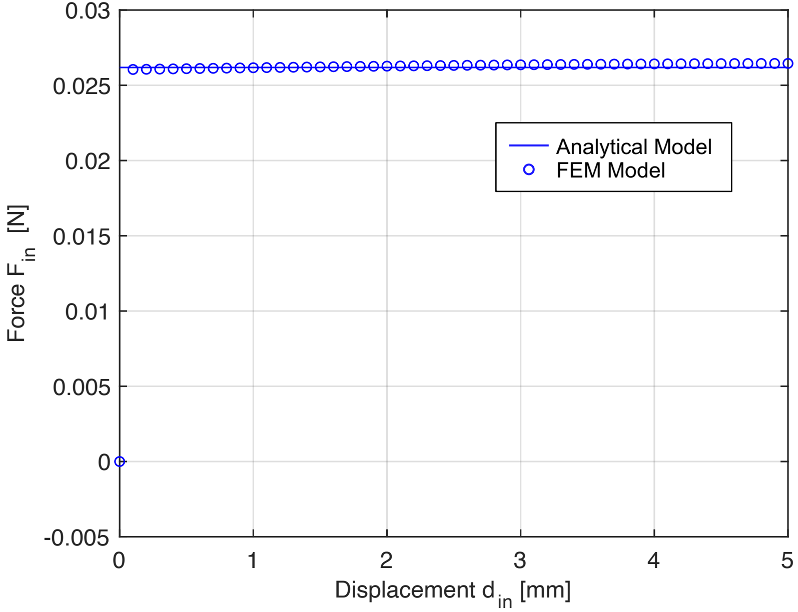
- L. Tissot-Daguette et al., **Load cell with adjustable stiffness based on a preloaded T-shaped flexure pivot**, in: Euspen's International Conference & Exhibition (2021)
- M. Smreczak et al., **A load cell with adjustable stiffness and zero offset tuning dedicated to electrical micro- and nanoprobing**, Precision Engineering, 76, 208-225 (2022)
- L. Tissot-Daguette et al., **Flexure pivot based system**, WO2022122629 (A1), Assignee: EPFL (2022)
- M. Kahrobaiyan et al., **Device for measuring a force exerted on an object**, EP3722767 (A1), Assignee: EPFL (2020)

Constant-force linear stage:

Preloading Type IV:
Self-Preloading

$$F_{\text{in}}(d_{\text{in}}) \cong cst$$





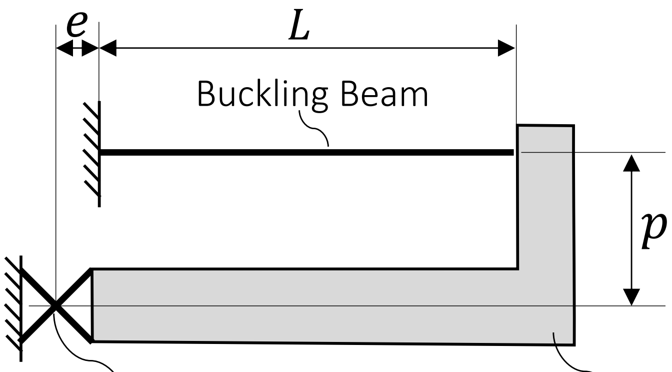
- L. Tissot-Daguette et al., **Three-beam constant-force flexure-based translation stage**, in press: Euspen's International Conference & Exhibition (2025)

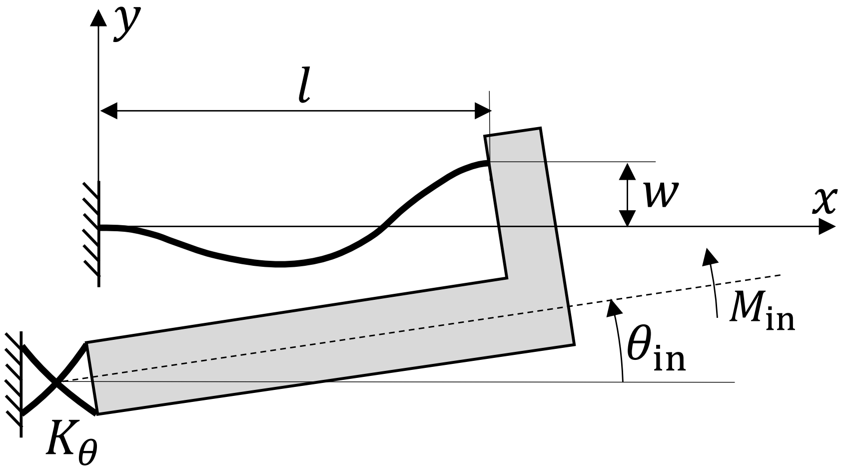
Constant-torque pivot:

Preloading Type IV: *Self-Preloading*

Patented & Published

$M_{\text{in}}(\theta_{\text{in}}) \cong cst$





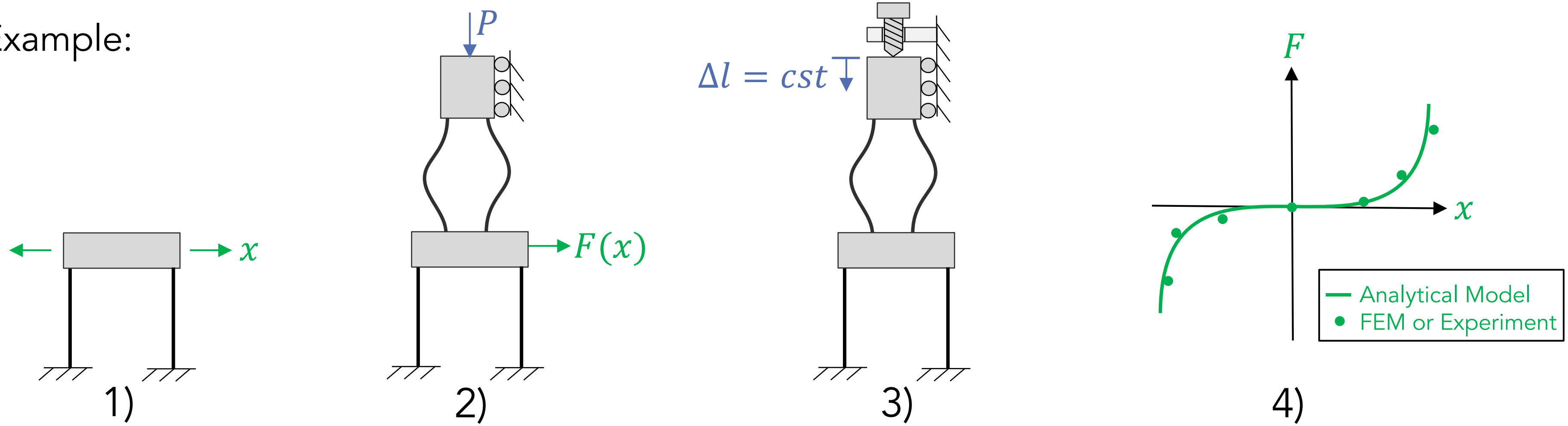
- L. Tissot-Daguette et al., **Design and modelling of a compliant constant-force surgical tool for objective assessment of ossicular chain mobility,** in: International Conference on Solid-State Sensors, Actuators and Microsystems (Transducers) (2021)
- L. Tissot-Daguette et al., **Detection mechanism for a medical sensing tool, medical sensing tool**, WO2022253808 (A1), Assignee: EPFL (2022)

Design methodology

Main phases of the design methodology:

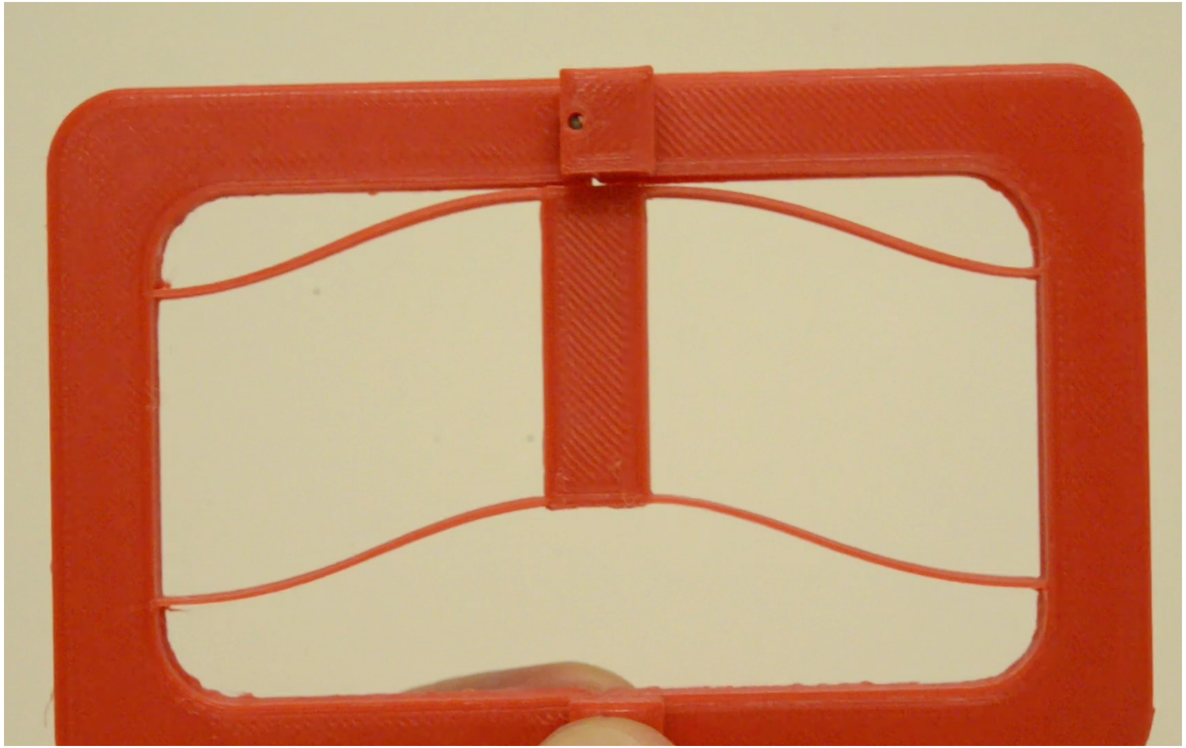
1) Design of the **guiding structure** of the mechanism to obtain the desired motion
2) Integration of **buckled beams** to tune the flexure mechanism's stiffness
3) Selection of the material, the manufacturing process and the **preloading type**
4) Validation of the design using **FEM and experiments**

Example:



Design guidelines:

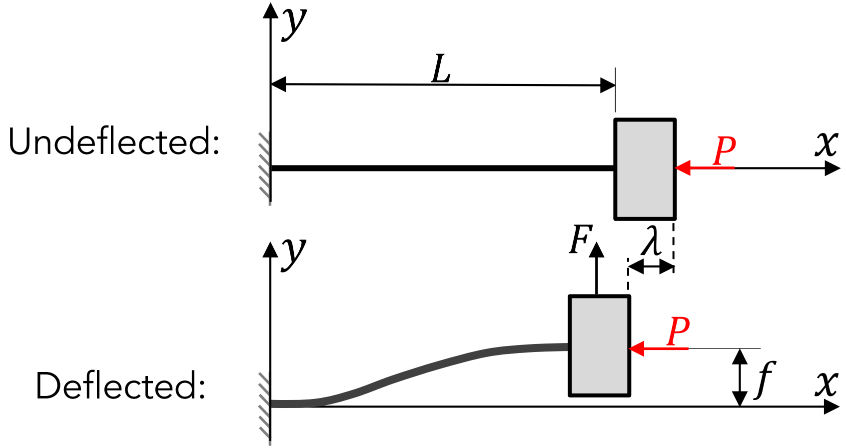
1. Buckling = Loss of Guidance
   - ➢ Separate buckling and guiding elements

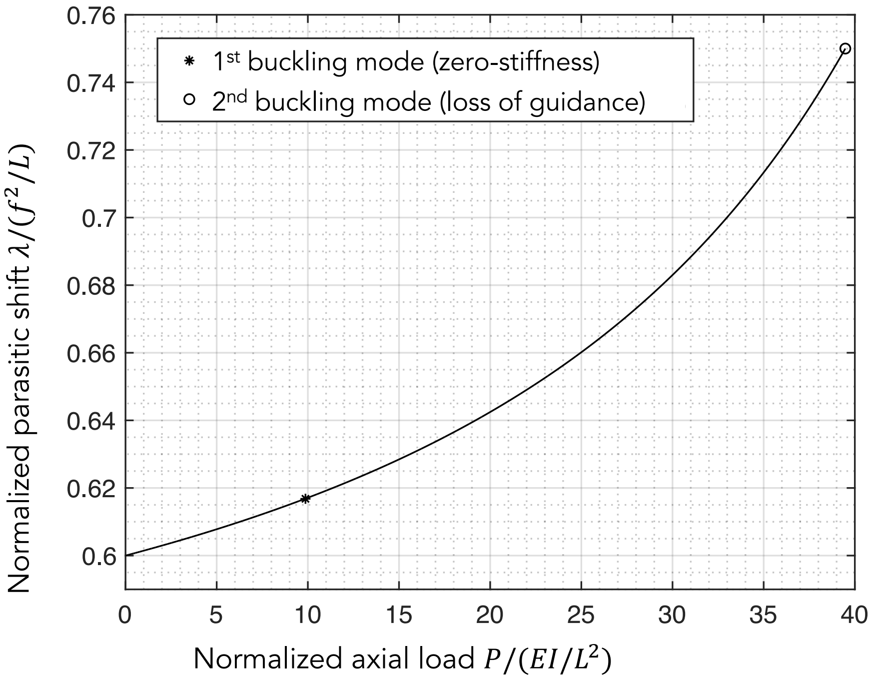


Design guidelines:

2. Parasitic shifts of flexures (λ) depend on the axial load (P)

➢ Consider the loads applied by the buckling elements onto the guiding elements



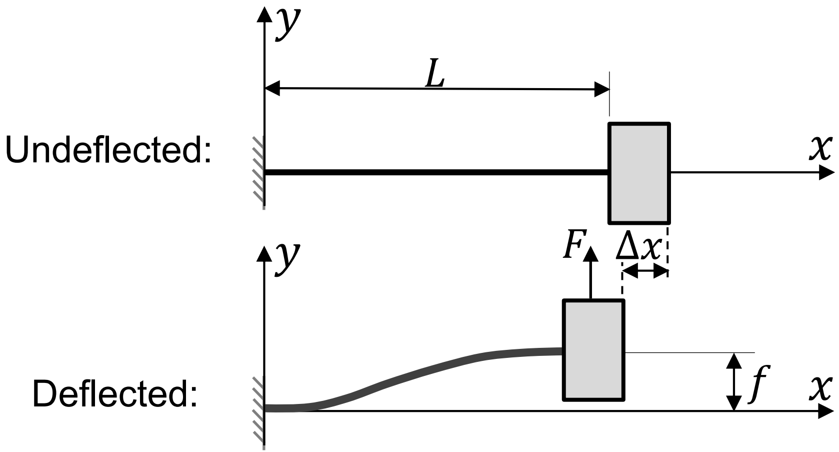


Generic design methodology: Guiding vs Buckling

- A beam buckling in second mode has a new DoF: Axial translation is free
- Buckling = Lost of guiding function

Generic design methodology: Parasitic shift modification

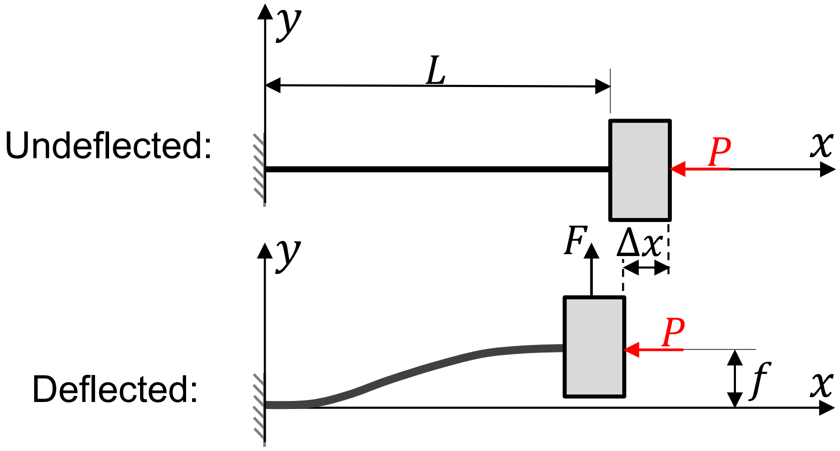
1) Translating Beam:

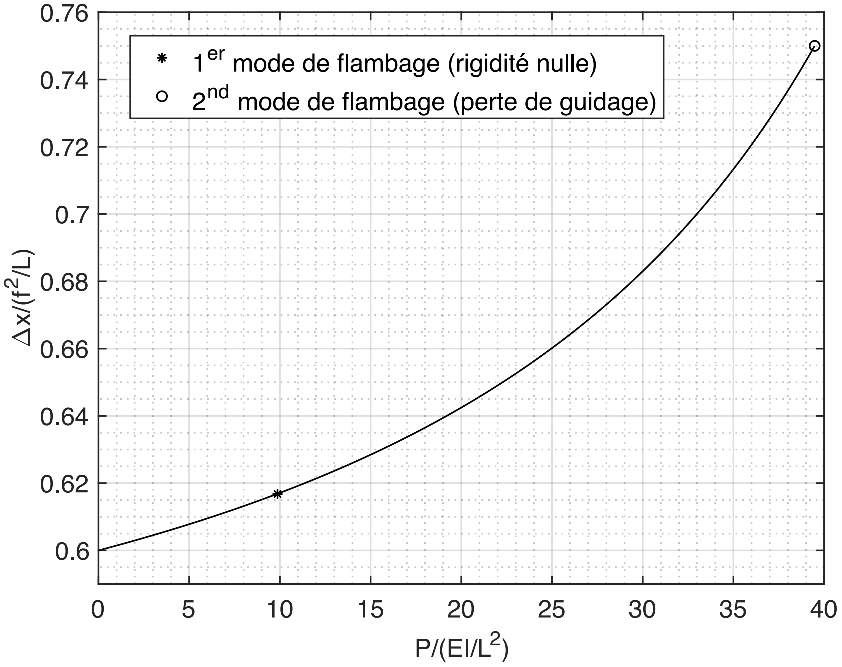


$$\Longrightarrow \boxed{\Delta x \cong \frac{3f^2}{5L}}$$

Generic design methodology: Parasitic shift modification

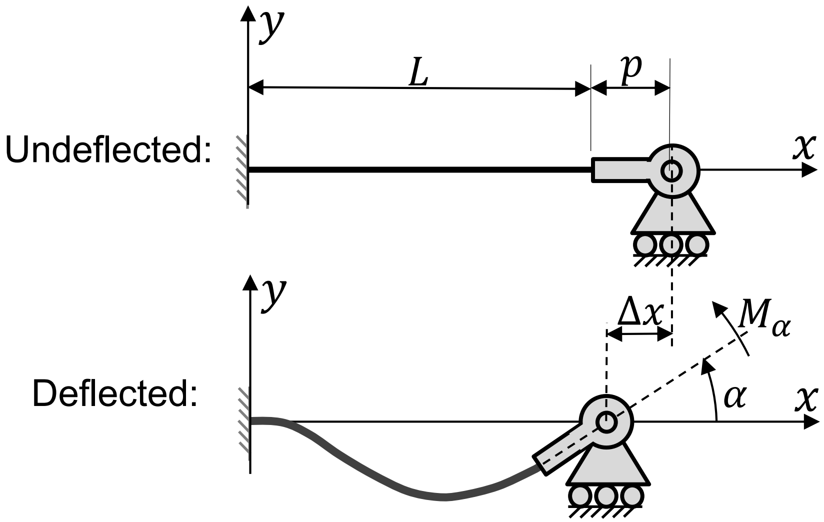
1) Translating Beam:





Generic design methodology: Parasitic shift modification

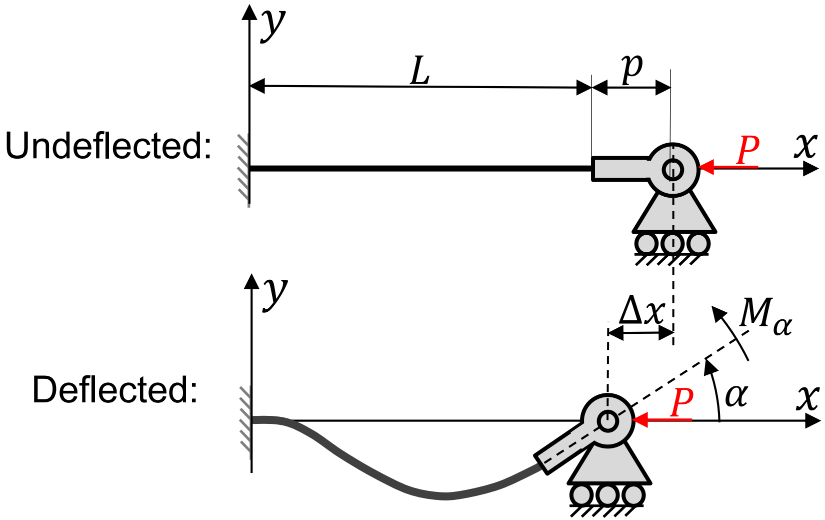
2) Rotating Beam:

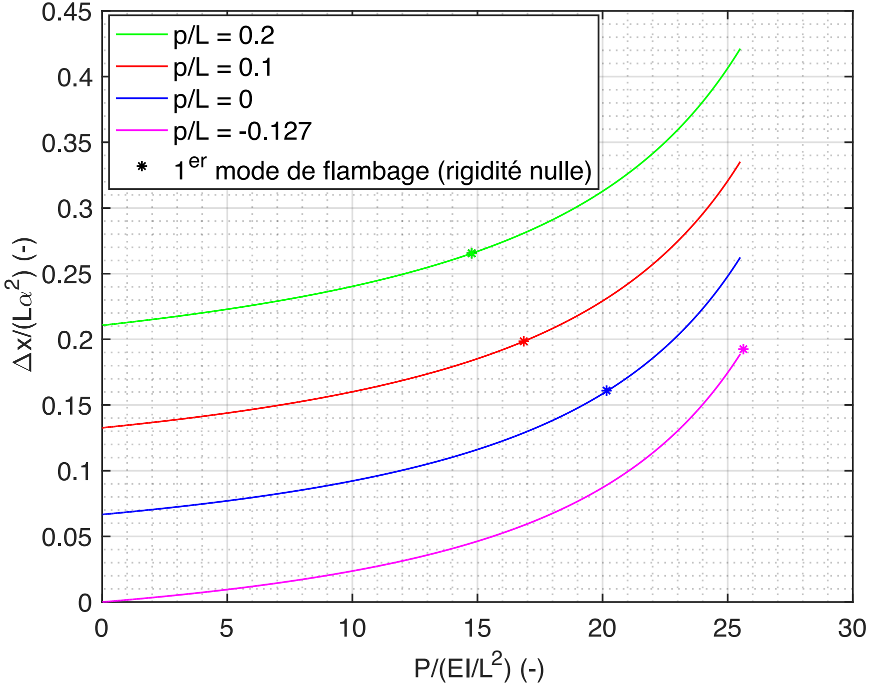


$$\Longrightarrow \boxed{\Delta x \cong \left(1 + 9\frac{p}{L} + 9\left(\frac{p}{L}\right)^2\right)\frac{L\alpha^2}{15}}$$

Generic design methodology: Parasitic shift modification

2) Rotating Beam:

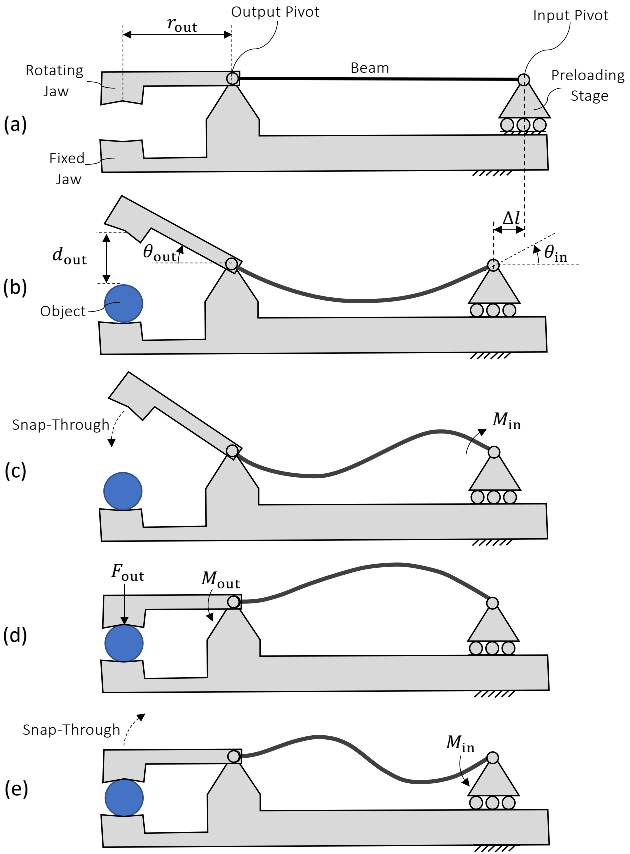




Application examples

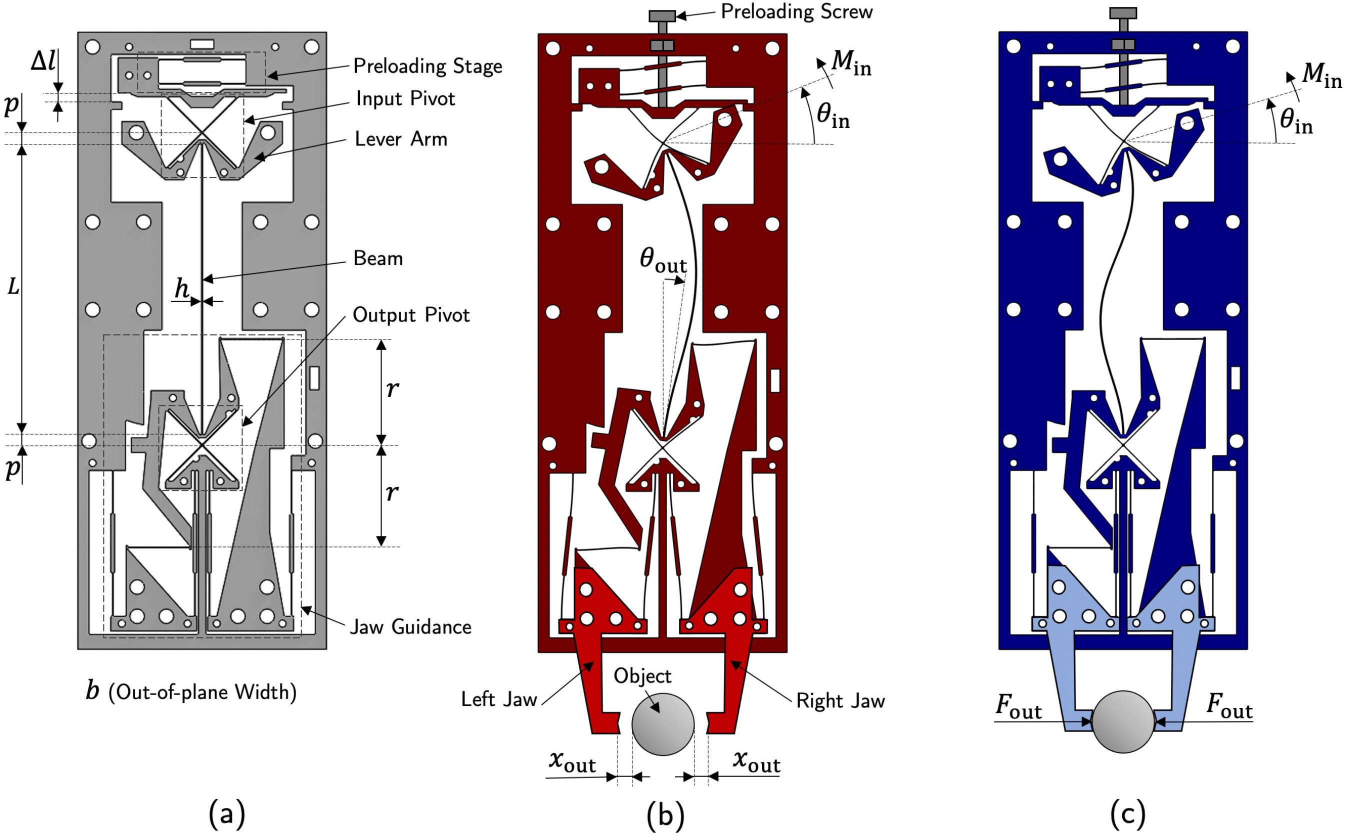
Bistable gripper: Concept

Preloading Type I: *Geometric*



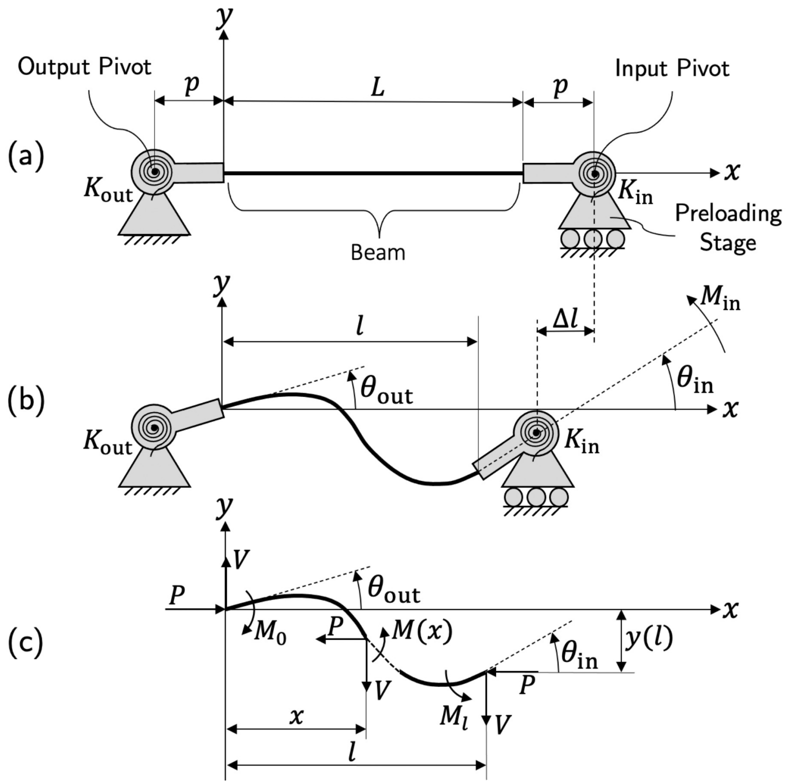
- No power is required from the actuator to keep the stable output states (open and closed)
- Very fast switching is obtained due to beam snap-through
- The output force is limited to a maximum value preventing damaging the object
- The preloading stage allows to adjust the opening stroke and the gripping force

Bistable gripper: Flexure Implementation

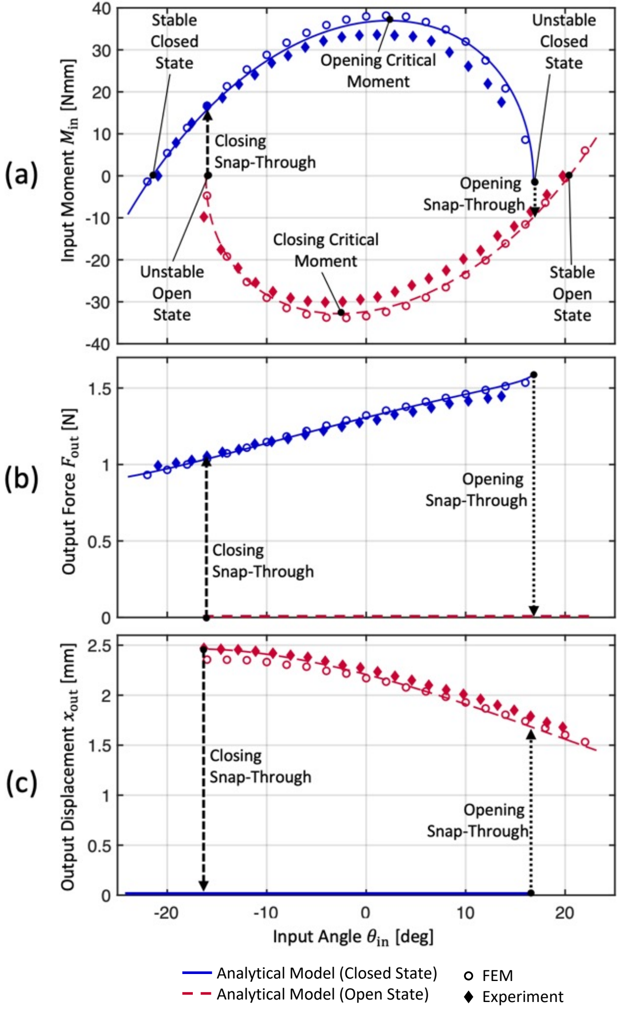


Bistable gripper as-fabricated (a), in open stable state (b) and in closed unstable state (c)

Bistable gripper: Modeling

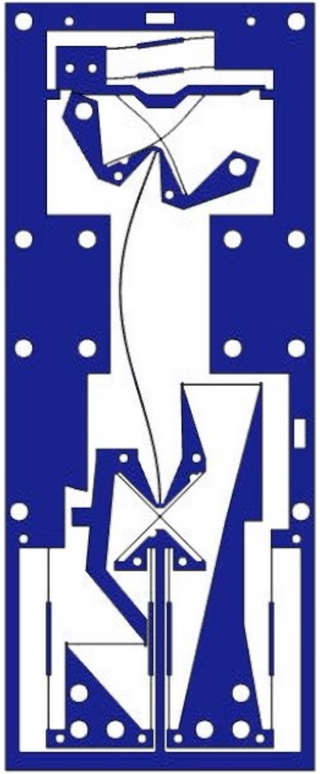


Modeling of the buckled beam, (a) as-fabricated, (b) deformed and (c) free-body diagram



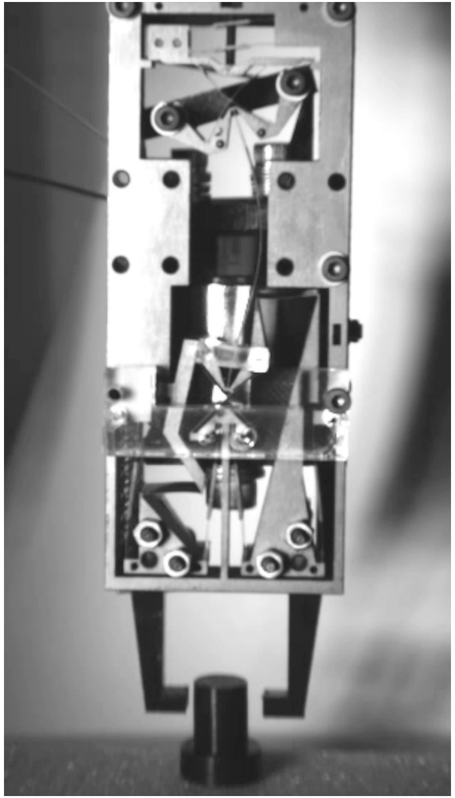
Bistable gripper: Results

Snap-Through	SMA Cooling
< 10 ms	> 5 s

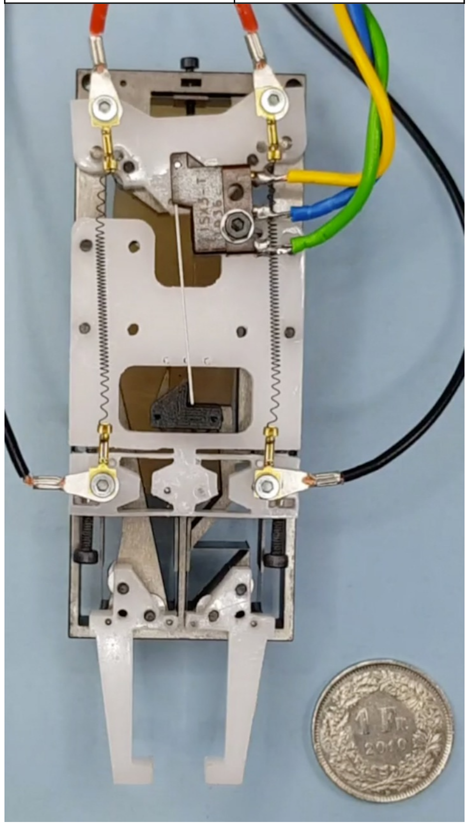


- FEM (Closed)
- FEM (Open)

EPFL Instant-Lab



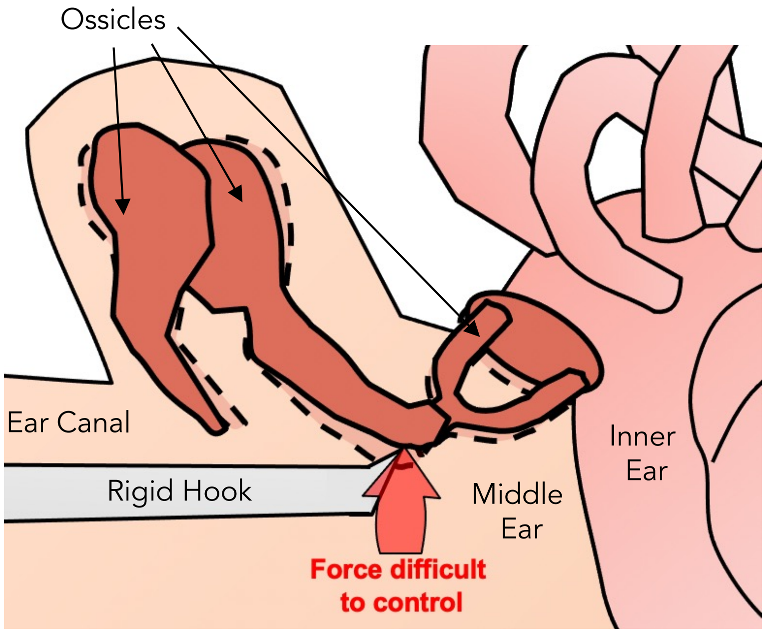
Prototype actuated by a voice coil (speed 0.006x).



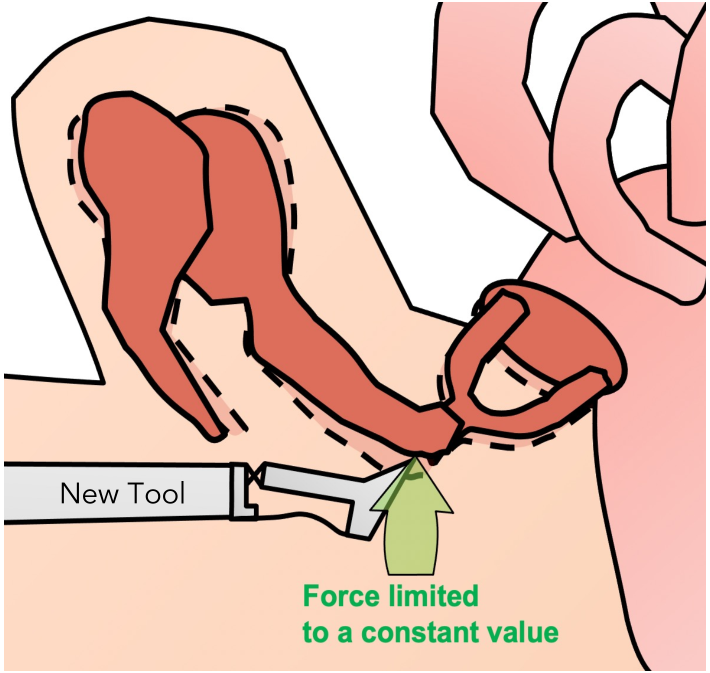
Prototype actuated by SMA springs (speed 2x). Credits: LAI, EPFL.

Constant-force surgical tool: Concept

Preloading Type IV:
Self-preloading

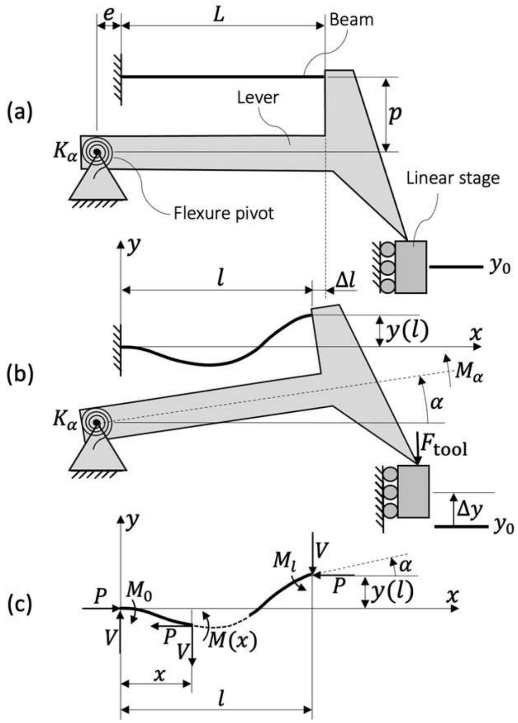


Common rigid hook to estimate the ossicle mobility

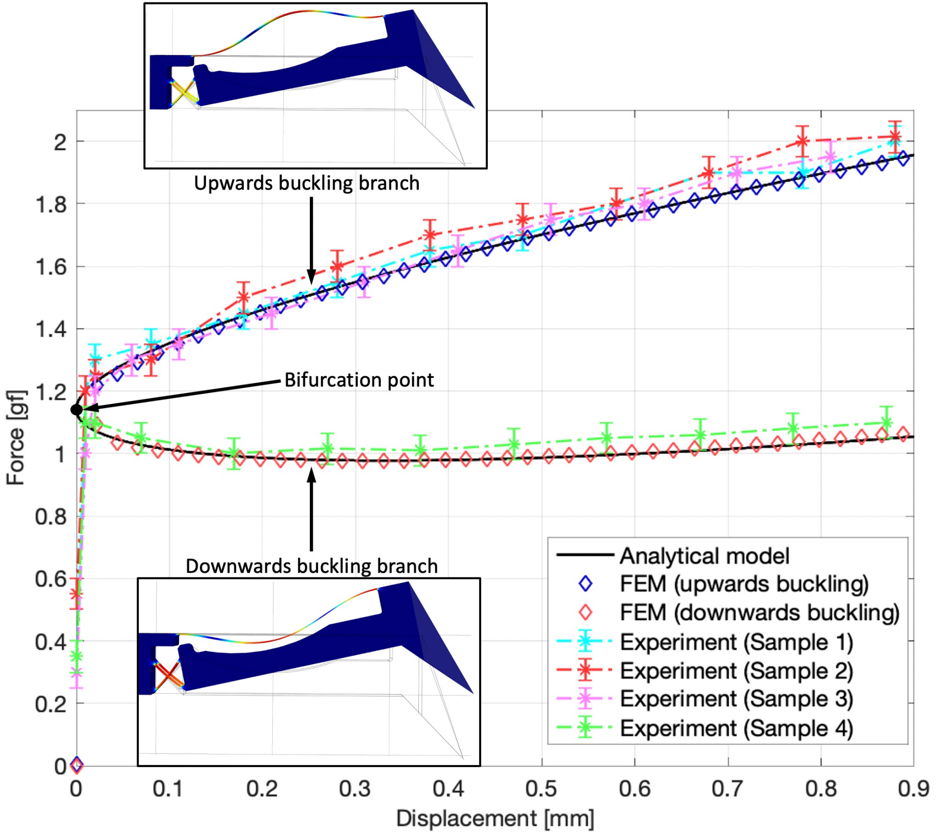


New tool to objectively assess the ossicle mobility

Constant-force surgical tool: Modeling

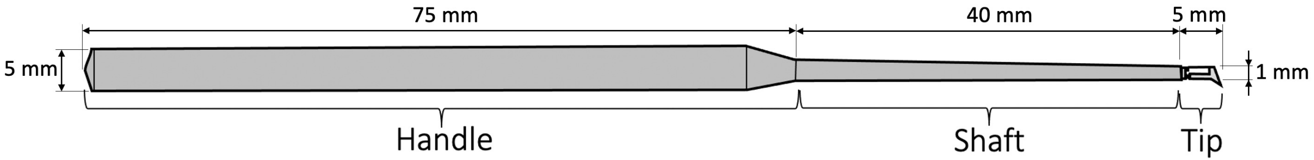


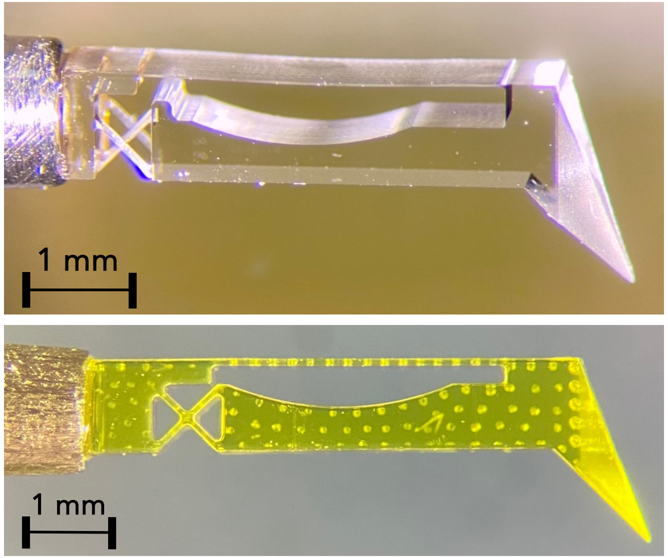
Modeling of the tool, (a) as-fabricated tool tip, (b) deformed tool tip and (c) free-body diagram of the beam



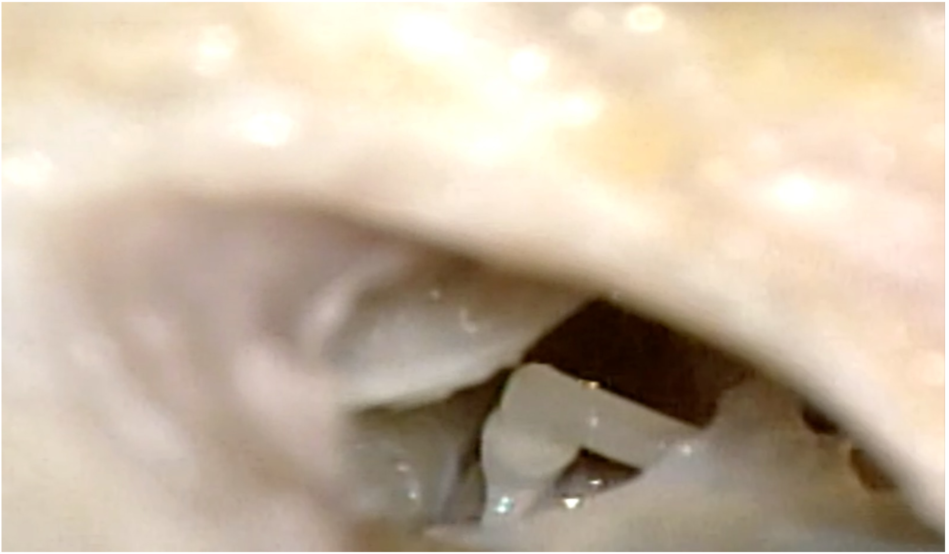
Force-displacement characteristics of the tool

Constant-force surgical tool: Prototype





The tip mechanism was fabricated in fused silica (top) and in resin (bottom).

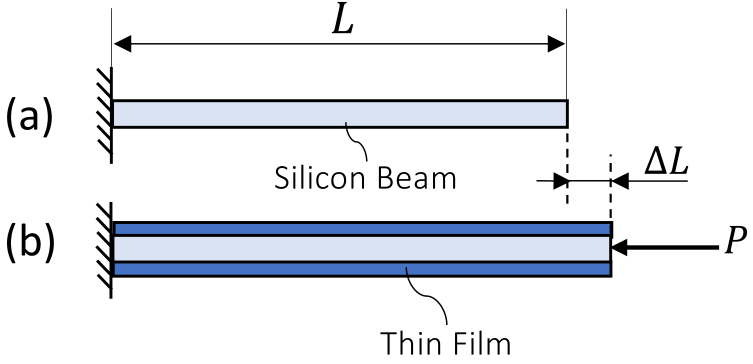


Video of the prototype testing. Credits: Universitäts Spital Zürich.

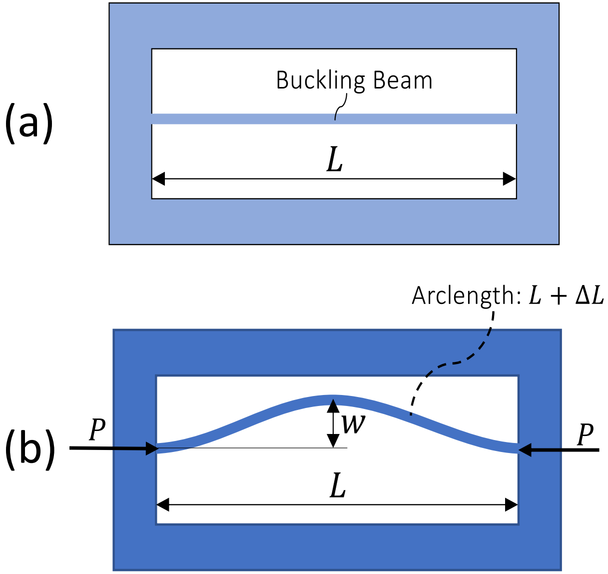
Thermally oxidized silicon mechanisms

Preloading Type V: *Residual stress*

Simple silicon beam preloaded by residual stress:



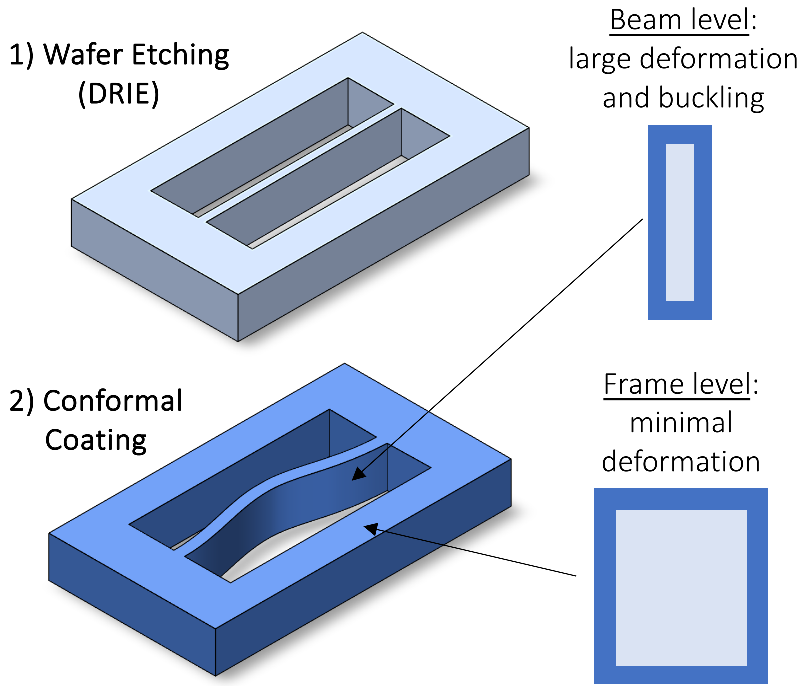
$$\frac{\Delta L}{L} \cong 0.1\%$$

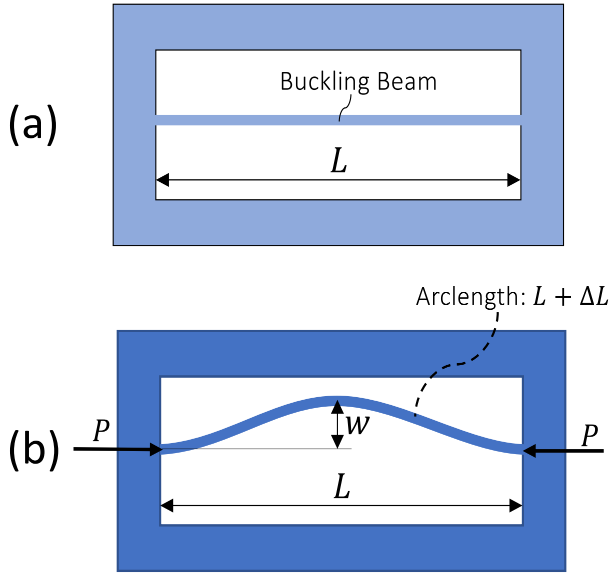


$$\frac{w}{L} \cong 1\%$$

Thermally oxidized silicon beam

Preloading Type V:
Residual Stress





$$\frac{w}{L} \cong 1\%$$

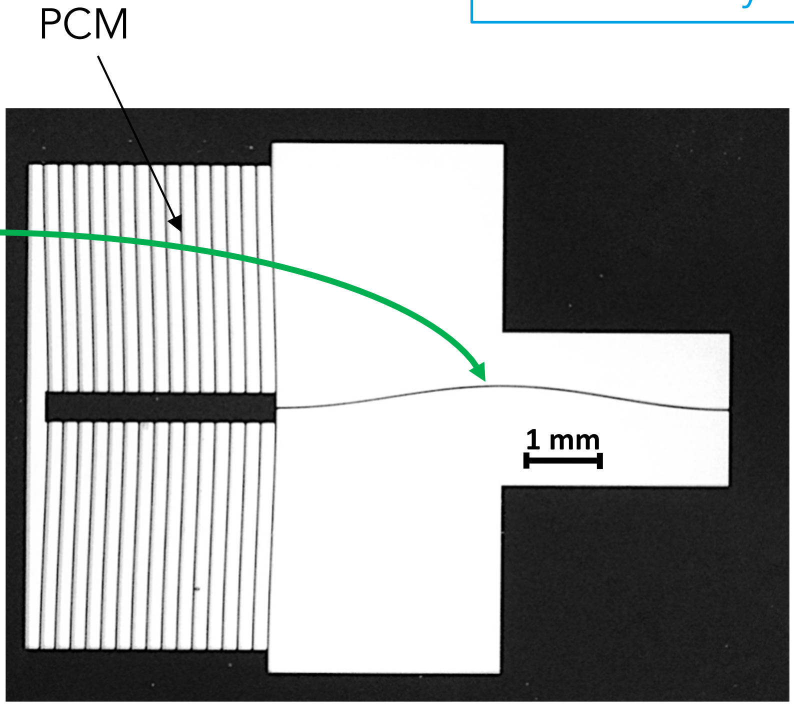
Preloading Chevron Mechanism

Preloading Type V: *Residual Stress*

Patented & Published

Fabrication by CSEM

Simple buckled beam



PCM-preloaded buckled beam

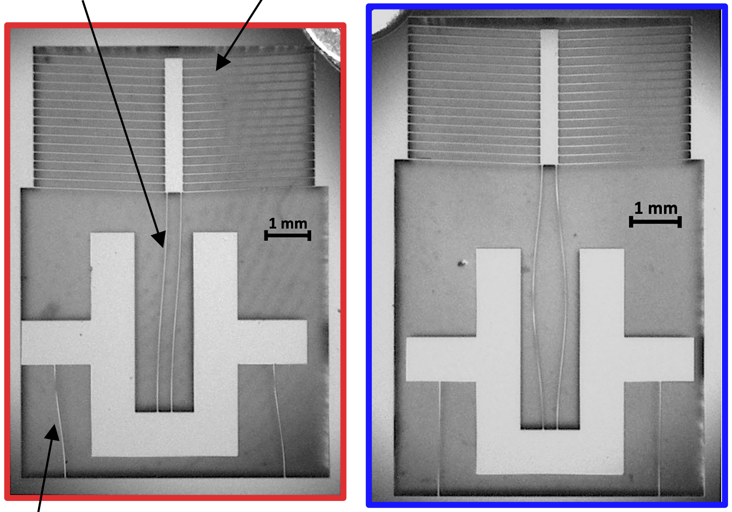
- L. Tissot-Daguette et al., Residual Stress Chevron Preloading Amplifier for Large-Stroke Stiffness Reduction of Silicon Flexure Mechanisms, J. Micromech. Microeng. 35, 025003 (2025)
- **Prestressed chevron structures for stiffness reduction of flexure mechanisms,** EPFL-TTO Licensing Opportunity, Nr 6.2564 (4.10.2024)

PCM-preloaded linear stage

Preloading Type V: *Residual stress*

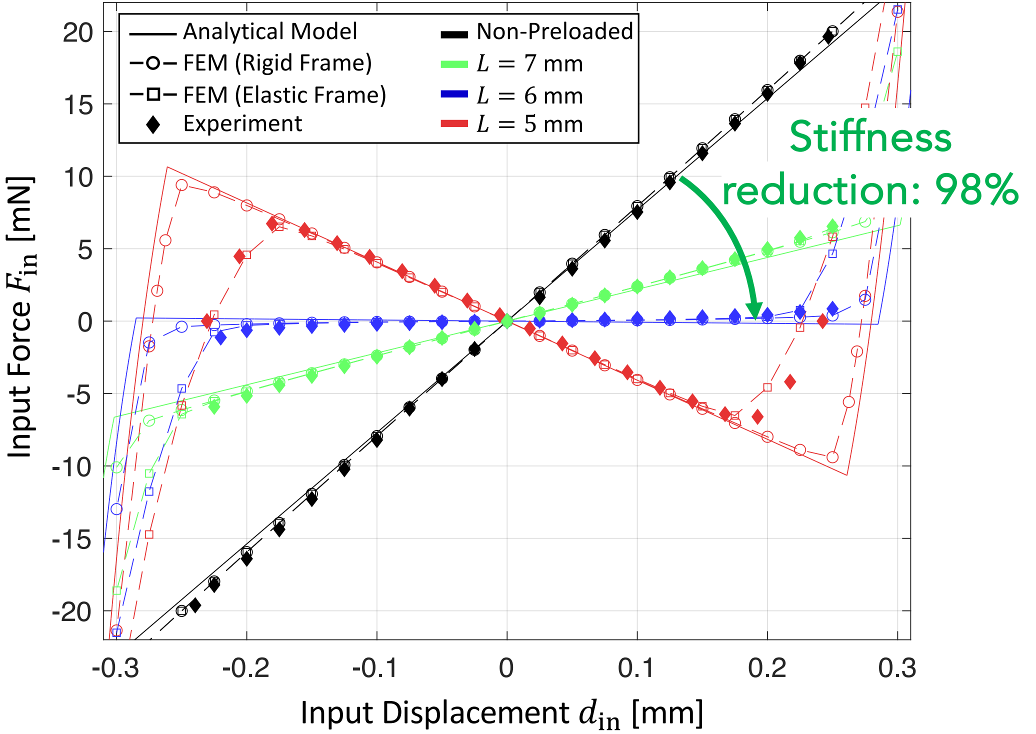
Patented & Published

Fabrication by CSEM



Negative stiffness (bistable)

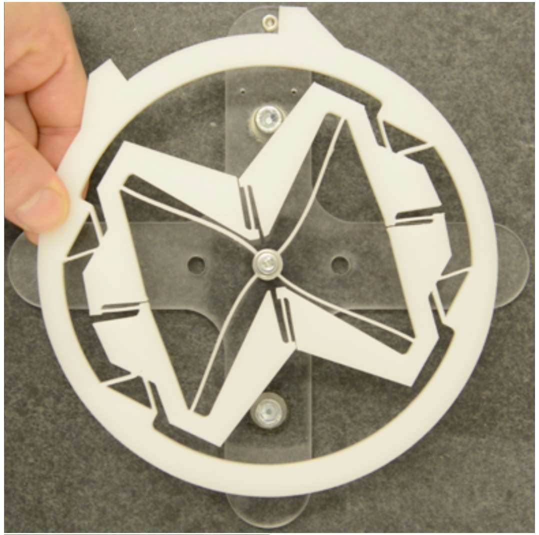
Near-zero positive stiffness (monostable)

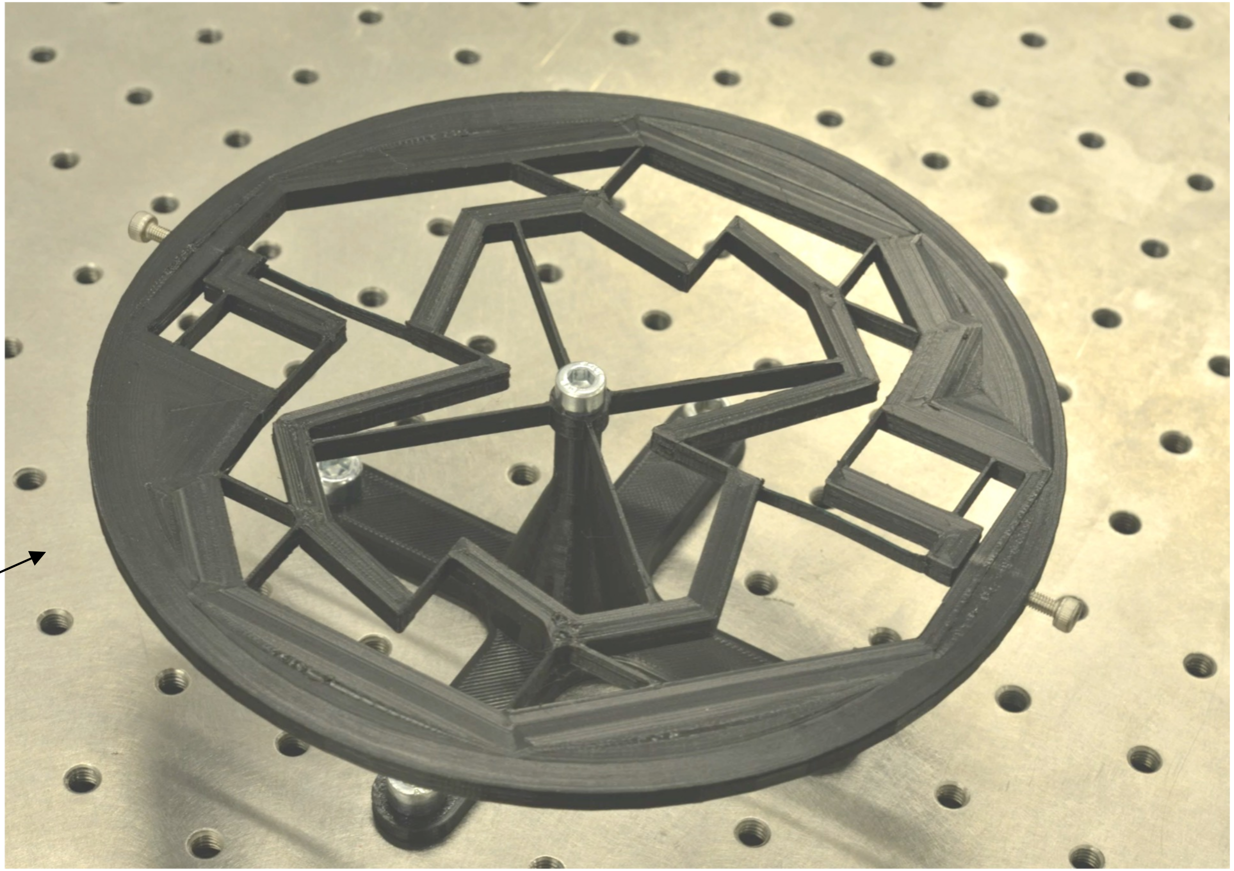


- L. Tissot-Daguette et al., Residual Stress Chevron Preloading Amplifier for Large-Stroke Stiffness Reduction of Silicon Flexure Mechanisms, J. Micromech. Microeng. 35, 025003 (2025)
- **Prestressed chevron structures for stiffness reduction of flexure mechanisms,** EPFL-TTO Licensing Opportunity, Nr 6.2564 (4.10.2024)

Thermally oxidized silicon mechanisms: Perspectives

"Silicon QUADRIVOT oscillator with reduced frequency"





Conclusion

1. Buckled beams can provide advantageous behaviors:
   - ✓ Bistability
   - ✓ Constant-force
   - ✓ Stiffness tuning
   - ✓ …
2. Design considerations:
   - ➢ Buckling vs guiding
   - ➢ Parasitic shift modification
3. Residual stress can be efficient to preload micro-mechanisms

References: Instant-Lab publications

[1] L. Tissot-Daguette, H. Schneegans, E. Thalmann, S. Henein
Analytical modeling and experimental validation of rotationally actuated pinned–pinned and fixed–pinned buckled beam bistable mechanisms
Mechanism and Machine Theory, Volume 174, August 2022, 104874

[2] L. Tissot-Daguette, M. Smreczak, C. Baur, S. Henein
Load cell with adjustable stiffness based on a preloaded T-shaped flexure pivot
Proceedings of the 21st EUSPEN International Conference, Copenhagen, June 2021

[3] M. Smreczak, L. Tissot-Daguette, E. Thalmann, C. Baur, S. Henein
A load cell with adjustable stiffness and zero offset tuning dedicated to electrical micro- and nanoprobing
Precision Engineering, Volume 76, July 2022, pp. 208-225

[4] L. Tissot-Daguette, C. Baur, A. Bertholds, P. Llosas and S. Henein
Design and modelling of a constant-force surgical tool for objective assessment of ossicular chain mobility
Transducers 2021 Virtual Conference, June 2021

[5] L. Tissot-Daguette, H. Schneegans, Q. Gubler, C. Baur, S. Henein
Rectilinear translation four-bar flexure mechanism based on four Remote Center Compliance pivots
Proceedings of the 22nd EUSPEN International Conference, Geneva, June 2022

[6] L. Tissot-Daguette, E. Thalmann, F. Cosandier, S. Henein **Zero parasitic shift pivoting kinematic structures based on coupled N-RRR planar parallel mechanisms for flexure pivot design**
IDETC, ASME, Boston, USA, August 2023

[7] L. Tissot-Daguette, F. Cosandier, E. Thalmann, S. Henein **Near-Zero Parasitic Shift Flexure Pivots Based on Coupled n-RRR Planar Parallel Mechanisms**
Journal of Mechanisms and Robotics, Volume 16(11), 2024, 111006